

Supplementary Material of the paper On robustness for spatio-temporal data

VOM+SAD approximation (10):

```
distribution<-function(e,n,a,va,g)
{
A<-1-pchisq(a*n/va,n)
B<-va/( sqrt(pi) *(a-va) )
C<-exp(-n*0.5*( (a/va)-1-log(a/va) ))
D<-sqrt(va)/sqrt(a-a*g^2+(g^2)*va) -1
resul<-A+e*sqrt(n)*B*C*D
return(resul)
}
```

where

```
va = 2gamma(h,tau) = spatio-temporal variogram
n = n(h,tau)
```

```
#####
```

```
"Exact" values by simulation
```

```
simu11<-function(n,B,e,va,g,a)
{
s<-NULL
for(j in 1:B){
s[j]<-mean((1-e)*va*rchisq(n,1)+e*g^2*va*rchisq(n,1))
}
P=ecdf(s)
1-P(a)
}
```

```
#####
```

```
Table 1
```

Approximations:

```
> distribution(0.01,3,2.5,1.4,1.1)
[1] 0.1482988
> distribution(0.01,3,3,1.4,1.1)
[1] 0.09323288
```

```

> distribution(0.01,3,3.5,1.4,1.1)
[1] 0.05812367
> distribution(0.01,3,4,1.4,1.1)
[1] 0.03600603
> distribution(0.01,3,4.5,1.4,1.1)
[1] 0.02219628
> distribution(0.01,3,5,1.4,1.1)
[1] 0.01363265

```

Exact:

```

> simu11(3,100000,0.01,1.4,1.1,2.5)
[1] 0.14714
> simu11(3,100000,0.01,1.4,1.1,3)
[1] 0.09308
> simu11(3,100000,0.01,1.4,1.1,3.5)
[1] 0.05577
> simu11(3,100000,0.01,1.4,1.1,4)
[1] 0.03548
> simu11(3,100000,0.01,1.4,1.1,4.5)
[1] 0.02089
> simu11(3,100000,0.01,1.4,1.1,5)
[1] 0.01313

```

#####

Figure 1

```

dibu1<-function(n,B,e,va,g)
{
s<-NULL
for(j in 1:B){
s[j]<-mean((1-e)*va*rchisq(n,1)+e*g^2*va*rchisq(n,1))
}
P=ecdf(s)
t<-seq(0,2.1,len=100)
plot(t,1-P(t),type="l",lwd=2,ylab=" ", xlab="a")
lines(t,approx(e,n,t,va,g),type="l",lty=2,ylab=" ",lwd=2,col=2)
}

```

```

dibu1(3,100000,0.01,1.4,1.1)

```

#####

Figure 2

```
dibu2<-function(n,B,e,va,g)
{
s<-NULL
for(j in 1:B){
s[j]<-mean((1-e)*va*rchisq(n,1)+e*g^2*va*rchisq(n,1))
}
P=ecdf(s)
t<-seq(3,5.5,len=100)
plot(t,1-P(t),type="l",lwd=2,ylab=" ",xlab="a")
}

dibu2(3,100000,0,1.4,1.1)

t<-seq(3,5.5,len=100)

lines(t,distribution(0.01,3,t,1.4,1.1),type="l",lty=2,ylab=" ",lwd=2,col=3)
lines(t,distribution(0.05,3,t,1.4,1.1),type="l",lty=3,ylab=" ",lwd=2,col=4)
lines(t,distribution(0.1,3,t,1.4,1.1),type="l",lty=4,ylab=" ",lwd=2,col=5)
lines(t,distribution(0.2,3,t,1.4,1.1),type="l",lty=5,ylab=" ",lwd=2,col=6)

legend(4.5, 0.06, legend=c("epsilon=0.01", "epsilon=0.05", "epsilon=0.1", "epsilon=0.2")
      col=c(3,4,5,6), lty=2:5, cex=0.8, lwd=2)

#####
```

Example

If `vv` is the object (p. 72 in Wikle et al., 2019).

```
library("CCA")
library("dplyr")
library("ggplot2")
```

```
library("gstat")
library("sp")
library("spacetime")
library("STRbook")

data("STObj3", package = "STRbook")
STObj4 <- STObj3[, "1993-07-01::1993-07-31"]

vvv <- variogram(object = z~1 + lat,
  data = STObj4,
  cloud=T,
  width = 80,
  cutoff = 1000,
  tlags = 0.01:6.01)
```

TWO-DIMENSIONAL Huber's spatio-temporal semivariogram estimator (with tuning constant equal to 1.345) of daily Tmax from the NOAA data set during July 2003, computed using the estimator introduced in Section 6 of the paper

```
library(MASS)
```

```
w1<-0
w2<-huber(vvv[vvv$dist > 0 & vvv$dist< 80 & vvv$timelag== 0.01,]$gamma,1.345)$mu
w3<-huber(vvv[vvv$dist > 80 & vvv$dist< 160 & vvv$timelag== 0.01,]$gamma,1.345)$mu
w4<-huber(vvv[vvv$dist > 160 & vvv$dist< 240 & vvv$timelag== 0.01,]$gamma,1.345)$mu
w5<-huber(vvv[vvv$dist > 240 & vvv$dist< 320 & vvv$timelag== 0.01,]$gamma,1.345)$mu
w6<-huber(vvv[vvv$dist > 320 & vvv$dist< 400 & vvv$timelag== 0.01,]$gamma,1.345)$mu
w7<-huber(vvv[vvv$dist > 400 & vvv$dist< 480 & vvv$timelag== 0.01,]$gamma,1.345)$mu
w8<-huber(vvv[vvv$dist > 480 & vvv$dist< 560 & vvv$timelag== 0.01,]$gamma,1.345)$mu
w9<-huber(vvv[vvv$dist > 560 & vvv$dist< 640 & vvv$timelag== 0.01,]$gamma,1.345)$mu
w10<-huber(vvv[vvv$dist > 640 & vvv$dist< 720 & vvv$timelag== 0.01,]$gamma,1.345)$mu
w11<-huber(vvv[vvv$dist > 720 & vvv$dist< 800 & vvv$timelag== 0.01,]$gamma,1.345)$mu
w12<-huber(vvv[vvv$dist > 800 & vvv$dist< 880 & vvv$timelag== 0.01,]$gamma,1.345)$mu
w13<-huber(vvv[vvv$dist > 880 & vvv$timelag== 0.01,]$gamma,1.345)$mu

w14<-huber(vvv[vvv$dist == 0 & vvv$timelag== 1.01,]$gamma,1.345)$mu
w15<-huber(vvv[vvv$dist > 0 & vvv$dist< 80 & vvv$timelag== 1.01,]$gamma,1.345)$mu
w16<-huber(vvv[vvv$dist > 80 & vvv$dist< 160 & vvv$timelag== 1.01,]$gamma,1.345)$mu
w17<-huber(vvv[vvv$dist > 160 & vvv$dist< 240 & vvv$timelag== 1.01,]$gamma,1.345)$mu
w18<-huber(vvv[vvv$dist > 240 & vvv$dist< 320 & vvv$timelag== 1.01,]$gamma,1.345)$mu
w19<-huber(vvv[vvv$dist > 320 & vvv$dist< 400 & vvv$timelag== 1.01,]$gamma,1.345)$mu
w20<-huber(vvv[vvv$dist > 400 & vvv$dist< 480 & vvv$timelag== 1.01,]$gamma,1.345)$mu
w21<-huber(vvv[vvv$dist > 480 & vvv$dist< 560 & vvv$timelag== 1.01,]$gamma,1.345)$mu
w22<-huber(vvv[vvv$dist > 560 & vvv$dist< 640 & vvv$timelag== 1.01,]$gamma,1.345)$mu
w23<-huber(vvv[vvv$dist > 640 & vvv$dist< 720 & vvv$timelag== 1.01,]$gamma,1.345)$mu
w24<-huber(vvv[vvv$dist > 720 & vvv$dist< 800 & vvv$timelag== 1.01,]$gamma,1.345)$mu
w25<-huber(vvv[vvv$dist > 800 & vvv$dist< 880 & vvv$timelag== 1.01,]$gamma,1.345)$mu
w26<-huber(vvv[vvv$dist > 880 & vvv$timelag== 1.01,]$gamma,1.345)$mu

w27<-huber(vvv[vvv$dist == 0 & vvv$timelag== 2.01,]$gamma,1.345)$mu
w28<-huber(vvv[vvv$dist > 0 & vvv$dist< 80 & vvv$timelag== 2.01,]$gamma,1.345)$mu
w29<-huber(vvv[vvv$dist > 80 & vvv$dist< 160 & vvv$timelag== 2.01,]$gamma,1.345)$mu
w30<-huber(vvv[vvv$dist > 160 & vvv$dist< 240 & vvv$timelag== 2.01,]$gamma,1.345)$mu
w31<-huber(vvv[vvv$dist > 240 & vvv$dist< 320 & vvv$timelag== 2.01,]$gamma,1.345)$mu
w32<-huber(vvv[vvv$dist > 320 & vvv$dist< 400 & vvv$timelag== 2.01,]$gamma,1.345)$mu
w33<-huber(vvv[vvv$dist > 400 & vvv$dist< 480 & vvv$timelag== 2.01,]$gamma,1.345)$mu
w34<-huber(vvv[vvv$dist > 480 & vvv$dist< 560 & vvv$timelag== 2.01,]$gamma,1.345)$mu
w35<-huber(vvv[vvv$dist > 560 & vvv$dist< 640 & vvv$timelag== 2.01,]$gamma,1.345)$mu
w36<-huber(vvv[vvv$dist > 640 & vvv$dist< 720 & vvv$timelag== 2.01,]$gamma,1.345)$mu
w37<-huber(vvv[vvv$dist > 720 & vvv$dist< 800 & vvv$timelag== 2.01,]$gamma,1.345)$mu
w38<-huber(vvv[vvv$dist > 800 & vvv$dist< 880 & vvv$timelag== 2.01,]$gamma,1.345)$mu
w39<-huber(vvv[vvv$dist > 880 & vvv$timelag== 2.01,]$gamma,1.345)$mu
```



```
w87<-huber(vvv[vvv$dist > 560 & vvv$dist< 640 & vvv$timelag== 6.01,]$gamma,1.345)$mu
w88<-huber(vvv[vvv$dist > 640 & vvv$dist< 720 & vvv$timelag== 6.01,]$gamma,1.345)$mu
w89<-huber(vvv[vvv$dist > 720 & vvv$dist< 800 & vvv$timelag== 6.01,]$gamma,1.345)$mu
w90<-huber(vvv[vvv$dist > 800 & vvv$dist< 880 & vvv$timelag== 6.01,]$gamma,1.345)$mu
w91<-huber(vvv[vvv$dist > 880 & vvv$timelag== 6.01,]$gamma,1.345)$mu
```

```
w<-c(w1,w2,w3,w4,w5,w6,w7,w8,w9,w10,w11,w12,w13,w14,w15,w16,w17,w18,w19,w20,w21,
w22,w23,w24,w25,w26,w27,w28,w29,w30,w31,w32,w33,w34,w35,w36,w37,w38,w39,w40,w41,
w42,w43,w44,w45,w46,w47,w48,w49,w50,w51,w52,w53,w54,w55,w56,w57,w58,w59,w60,w61,
w62,w63,w64,w65,w66,w67,w68,w69,w70,w71,w72,w73,w74,w75,w76,w77,w78,w79,w80,w81,
w82,w83,w84,w85,w86,w87,w88,w89,w90,w91)
```

```
huberst<-vv
huberst$gamma<-w
```

Figure 3 of the paper

```
plot(huberst,main="Huber's spatio-temporal semivariogram estimator")
```

THREE-DIMENSIONAL Classical and Hurber's spatio-temporal semivariogram estimators

Classical

Figure 4 of the paper

```
plot(vv, all=T, wireframe=T, zlim=c(0,30),zlab=NULL,main="Classical spatio-temporal  
semivariogram estimator of daily maximum temperatures", xlab=list("distance (km)",  
rot=30),ylab=list("time lag (days)", rot=-35),scales=list(arrows=F, z = list(distance = 5)))
```

Robust

Figure 5 of the paper

```
plot(huberst, all=T, wireframe=T, zlim=c(0,30),zlab=NULL,  
main="Huber's spatio-temporal semivariogram estimator",  
xlab=list("distance (km)", rot=30),ylab=list("time lag (days)", rot=-35),  
scales=list(arrows=F, z = list(distance = 5)))
```

Figure 6 of the paper

```
plot(vv,map=F)
```

Figure 7 of the paper

```
plot(huberst,map=F)
```



```

va= 2gamma(h,t)    initial t
a= 2gamma(h,t+tau)  posterior t+tau

valuetime<-function(n,va,a)
{
A<-1-pchisq(a*n/va,n)
B<-va/( sqrt(pi) *(a-va) )
C<-exp(-n*0.5*( (a/va)-1-log(a/va) ))
D<-sqrt(a/va)*exp( -a/(2*va)+0.5 )
resul<-A+sqrt(n)*B*C*(D-1)
return(resul)
}

y5<-mean(vvv[vvv$dist > 240& vvv$dist< 320 & vvv$timelag== 0.01,]$gamma)
y18<-mean(vvv[vvv$dist > 240& vvv$dist< 320 & vvv$timelag== 1.01,]$gamma)
y31<-mean(vvv[vvv$dist > 240& vvv$dist< 320 & vvv$timelag== 2.01,]$gamma)
y57<-mean(vvv[vvv$dist > 240& vvv$dist< 320 & vvv$timelag== 4.01,]$gamma)
y70<-mean(vvv[vvv$dist > 240& vvv$dist< 320 & vvv$timelag== 5.01,]$gamma)
y83<-mean(vvv[vvv$dist > 240& vvv$dist< 320 & vvv$timelag== 6.01,]$gamma)

n=806 in all of them

```

Significant difference between the initial moment and the first temporal moment:

```
> valuetime(806,y5,y18)
[1] -1.053956e-12
```

or between the first and second moment:

```
> valuetime(806,y18,y31)
[1] -3.526265e-07
```

No significant differences between the fourth and fifth temporal lags

```
> valuetime(806,y57,y70)
[1] 0.9427521
```

or between the fifth and sixth temporal lags

```
> valuetime(806,y70,y83)
[1] 0.9737844
```

Identification of spatio-temporal outliers. Example 1.

2*vv= classical spatio-temporal estimator.

2*huberst= Huber's spatio-temporal estimator.

With

```
> 2*vv$gamma-2*huberst$gamma
[1] 0.00000000 0.11815214 0.05988491 0.04634051 0.27104432 0.30030481
 [7] 0.64103610 0.55708326 1.41763057 1.72616658 2.09372284 2.63389680
[13] 2.76044786 1.08615712 3.08811040 1.63682044 1.72247747 1.22742633
[19] 0.88295277 0.79972846 0.45464791 0.81244694 1.23359746 1.07838803
[25] 1.17471658 1.53057483 2.59012665 6.46893813 3.82553145 4.46647336
[31] 2.65997242 2.87686600 2.70273487 1.78644411 1.30851738 1.30007439
[37] 0.87995595 0.69687875 0.32447860 2.06136807 6.69595187 3.41120232
[43] 3.79548627 2.70105019 2.32587473 2.52065924 1.86054958 1.44473634
[49] 1.61053058 1.11151622 1.22499377 0.73239271 1.91869460 6.37045254
[55] 2.20228578 2.73033814 2.04338732 2.00811742 1.44673245 1.15043167
[61] 0.84946158 0.86724293 0.60570998 0.64352349 0.41799575 1.35191287
[67] 5.75647785 2.22999570 4.06605990 2.68697489 2.10983544 3.41917519
[73] 2.31835873 2.99840272 3.77751597 1.66521984 1.20132517 1.19066048
[79] 2.84824909 6.55972115 4.11515046 4.97258728 4.04917717 3.12657133
[85] 3.21807028 2.64135431 2.37752224 1.86357447 1.43937821 2.04703953
[91] 1.53870648
```

we obtain the differences (always positive) of these two M-estimators. Some are very small: 0.00000000, 0.05988491, ... Others are somewhat large: 3.79548627, 6.55972115, ... Which ones are significant?

Because the sample sizes are large, we shall use the normal approximation as we say in Remark 1,

$$P_F\{T_n > a\} \simeq 1 - \Phi((a - E[T_n])/\sigma_{T_n})$$

where $T_n = \frac{1}{n} \sum_{i=1}^{n(\mathbf{h}, \tau)} (X_i - \psi_b(X_i))$. In fact we can use the Central Limit Theorem to obtain the previous approximation.

Huber's spatio-temporal variogram estimator is biased for the spatio-temporal variogram $2\gamma_z(\mathbf{h}; \tau)$ because it is based on the squares X_u . Then, although the classical one is not biased, the M-estimator difference will be also biased being its expectation under F ,

$$F = (1 - \epsilon) 2\gamma_z(\mathbf{h}; \tau) \chi_1^2 + \epsilon g^2 2\gamma_z(\mathbf{h}; \tau) \chi_1^2$$

$$E[X_i - \psi_b(X_i)] = (1 - \epsilon) \left(2\gamma_z(\mathbf{h}; \tau) \left(1 - F_{\chi_3^2}(b/(2\gamma_z(\mathbf{h}; \tau))) \right) \right) - b \left(1 - F_{\chi_1^2}(b/(2\gamma_z(\mathbf{h}; \tau))) \right)$$

$$+\epsilon \left(g^2 2 \gamma_z(\mathbf{h}; \tau) \left(1 - F_{\chi_3^2}(b/(g^2 2 \gamma_z(\mathbf{h}; \tau))) \right) - b \left(1 - F_{\chi_1^2}(b/(g^2 2 \gamma_z(\mathbf{h}; \tau))) \right) \right).$$

Also it is

$$\begin{aligned} E[(X_i - \psi_b(X_i))^2] &= (1 - \epsilon) \left(3(2 \gamma_z(\mathbf{h}; \tau))^2 \left(1 - F_{\chi_3^2}(b/(2 \gamma_z(\mathbf{h}; \tau))) \right) \right. \\ &\quad \left. - 2b(2 \gamma_z(\mathbf{h}; \tau)) \left(1 - F_{\chi_3^2}(b/(2 \gamma_z(\mathbf{h}; \tau))) \right) \right) \\ &+ b^2 \left(1 - F_{\chi_1^2}(b/(2 \gamma_z(\mathbf{h}; \tau))) \right) + \epsilon \left(3(g^2 2 \gamma_z(\mathbf{h}; \tau))^2 \left(1 - F_{\chi_3^2}(b/(g^2 2 \gamma_z(\mathbf{h}; \tau))) \right) \right. \\ &\quad \left. - 2b(g^2 2 \gamma_z(\mathbf{h}; \tau)) \left(1 - F_{\chi_3^2}(b/(g^2 2 \gamma_z(\mathbf{h}; \tau))) \right) \right) \\ &\quad \left. + b^2 \left(1 - F_{\chi_1^2}(b/(g^2 2 \gamma_z(\mathbf{h}; \tau))) \right) \right) \end{aligned}$$

then, we can compute the tail probabilities

$$P_F\{T_n > a\} \simeq 1 - \Phi((a - E[T_n])/\sigma_{T_n})$$

for the points a , considering here that $\epsilon = 0.01$ and $g = 1.1$,

```
vario = 2*gamma
```

```
media<-function(b,vario)
```

```
{
A1<-0.99*(vario*(1-pchisq(b/vario,3))-b*(1-pchisq(b/vario,1)))+
0.01*((1.1)^2*vario*(1-pchisq(b/((1.1)^2*vario),3))-b*(1-pchisq(b/((1.1)^2*vario),1)))
return(A1)
}
```

```
variance<-function(b,vario)
```

```
{
A1<-0.99*(vario*(1-pchisq(b/vario,3))-b*(1-pchisq(b/vario,1)))+
0.01*((1.1)^2*vario*(1-pchisq(b/((1.1)^2*vario),3))-b*(1-pchisq(b/((1.1)^2*vario),1)))
A2<-0.99*( 3*((vario)^2)*(1-pchisq(b/vario,5))-(2*b*vario)*
(1-pchisq(b/vario,3))+(b^2)*(1-pchisq(b/vario,1)) ) + 0.01*
(3*((vario*(1.1)^2)^2)*(1-pchisq(b/(vario*(1.1)^2),5))
-(2*b*vario*(1.1)^2)*(1-pchisq(b/(vario*(1.1)^2),3))+(b^2)*(1-pchisq(b/(vario*(1.1)^2),1))
)
solu<-A2-(A1)^2
return(solu)
}
```

a = 2*vv\$gamma-2*huberst\$gamma		Tail probability	
[1] 0.0000000		1	
[2] 0.11815214		0.9696010	
[3] 0.05988491		0.9780117	
[4] 0.04634051		0.9790160	
[5] 0.27104432		0.9604991	
[6] 0.30030481		0.9578807	
[7] 0.64103610		0.9090644	
[8] 0.55708326		0.9205907	
[9] 1.41763057		0.6816292	
[10] 1.72616658		0.5576323	
[11] 2.09372284		0.4016904	
[12] 2.63389680		0.2052651	
[13] 2.76044786		0.1657873	
[14] 1.08615712		0.7490757	
[15] 3.08811040		0.0649727	
[16] 1.63682044		0.5550302	
[17] 1.72247747		0.5352132	
[18] 1.22742633		0.7489740	
[19] 0.88295277		0.8538907	
[20] 0.79972846		0.8692775	
[21] 0.45464791		0.9342956	
[22] 0.81244694		0.8671335	
[23] 1.23359746		0.7525291	
[24] 1.07838803		0.7960837	
[25] 1.17471658		0.7630998	
[26] 1.53057483		0.6353079	
[27] 2.59012665		0.1613022	
[28] 6.46893813		1.8235e-07	<-----
[29] 3.82553145		0.0139424	<-----
[30] 4.46647336		0.0022001	<-----
[31] 2.65997242		0.1725364	
[32] 2.87686600		0.1283164	
[33] 2.70273487		0.1740595	
[34] 1.78644411		0.5210816	
[35] 1.30851738		0.7077834	
[36] 1.30007439		0.7139480	
[37] 0.87995595		0.8430800	
[38] 0.69687875		0.8840712	
[39] 0.32447860		0.9412339	
[40] 2.06136807		0.3682308	
[41] 6.69595187		8.9457e-08	<-----
[42] 3.41120232		0.0423737	<-----
[43] 3.79548627		0.0179283	<-----
[44] 2.70105019		0.1767490	
[45] 2.32587473		0.2995524	
[46] 2.52065924		0.2341492	
[47] 1.86054958		0.4950422	
[48] 1.44473634		0.6594336	

[49]	1.61053058	0.5978700	
[50]	1.11151622	0.7796753	
[51]	1.22499377	0.7412523	
[52]	0.73239271	0.8740286	
[53]	1.91869460	0.4081195	
[54]	6.37045254	4.3693e-07	<-----
[55]	2.20228578	0.2916914	
[56]	2.73033814	0.1435263	
[57]	2.04338732	0.3807738	
[58]	2.00811742	0.4066451	
[59]	1.44673245	0.6285479	
[60]	1.15043167	0.7494908	
[61]	0.84946158	0.8359542	
[62]	0.86724293	0.8288259	
[63]	0.60570998	0.8905084	
[64]	0.64352349	0.8805237	
[65]	0.41799575	0.9191782	
[66]	1.35191287	0.6235374	
[67]	5.75647785	1.2212e-05	<-----
[68]	2.22999570	0.2857393	
[69]	4.06605990	0.0079754	<-----
[70]	2.68697489	0.1608218	
[71]	2.10983544	0.3477347	
[72]	3.41917519	0.0431910	<-----
[73]	2.31835873	0.2808030	
[74]	2.99840272	0.1028271	
[75]	3.77751597	0.0202341	<-----
[76]	1.66521984	0.5364928	
[77]	1.20132517	0.7115017	
[78]	1.19066048	0.7216741	
[79]	2.84824909	0.1096408	
[80]	6.55972115	1.5528e-07	<-----
[81]	4.11515046	0.0059802	<-----
[82]	4.97258728	0.0003689	<-----
[83]	4.04917717	0.0079846	<-----
[84]	3.12657133	0.0722094	
[85]	3.21807028	0.0584898	
[86]	2.64135431	0.1709064	
[87]	2.37752224	0.2434866	
[88]	1.86357447	0.4240598	
[89]	1.43937821	0.6023280	
[90]	2.04703953	0.3630105	
[91]	1.53870648	0.5682633	

For instance,

```
> 1-pnorm( (2*vv$gamma-2*huber$t$gamma)[52] - media(b52, 2*vv$gamma[52]) /
+ (sqrt( variance(b52 , 2*vv$gamma[52]) / length(datos52)) ) )
[1] 0.8740286
```

Figure 9 of the paper

```
x0<-c(0,40,120,200,280,360,440,520,600,680,760,840,920)
x<-(rep(x0,7))
y<-c( rep(0,13), rep(1,13),rep(2,13),rep(3,13),rep(4,13),rep(5,13),rep(6,13))
plot(x,y,xlab="spatial lags",ylab="temporal lags")
points(40,2,pch=16,col=2)
points(120,2,pch=16,col=2)
points(200,2,pch=16,col=2)
points(40,3,pch=16,col=2)
points(200,3,pch=16,col=2)
points(40,4,pch=16,col=2)
points(40,5,pch=16,col=2)
points(200,5,pch=16,col=2)
points(40,6,pch=16,col=2)
points(120,6,pch=16,col=2)
points(200,6,pch=16,col=2)
points(280,6,pch=16,col=2)
points(40,1,pch=16,col=4)
points(120,3,pch=16,col=4)
points(440,5,pch=16,col=4)
points(680,5,pch=16,col=4)
points(360,6,pch=16,col=4)
points(440,6,pch=16,col=4)
```