Abstract

In this paper, we provide an alternative representation for Fixed Capital to the commonly accepted one within the Sraffian literature (the Torrens rule). This new method for representing Fixed Capital calculates the profit rate using the total capital value displayed in the firm’s balance sheet at the beginning of each time period. This contrasts with the Torrens rule, which calculates the profit rate using the lump sum of circulating capital for purchasing material inputs plus the book value of fixed capital stock. The core of this paper is precisely to provide the rationale for the new procedure, which is grounded on an elementary knowledge of Financial Accounting. This new method gets rid of the fixed capital stock book values, so it is better adapted than the Torrens rule to show that the market prices along with the distributive variables perform the reproduction of economic system. At the same time, this new method provides us with an accounting foundation for the widespread practice within input-output analysis of combining the fixed capital stock valued as new with a proportional depreciation.

_J.E.L. classification:_ D24.

_Keywords:_ Fixed Capital; Sraffa; von Neumann.
1. INTRODUCTION

It is common place within the literature on Fixed Capital in the Sraffian Theory of Prices, that the general way to treat the former is to represent it in terms of joint production, wherein the same fixed capital good in successive phases of wear and tear is considered as a qualitatively differentiated commodity. See for instance Sraffa (1960, Chap. 10), Roncaglia (1971) (1978, pp. 36-48), Pasinetti (1973), Baldone (1974), Varri (1974), Schefold (1978)(1980) and Kurz and Salvadori (1995, Chaps. 7 and 9), among other authors. This treatment of Fixed Capital stems from the Classical tradition, as reflected in Sraffa (1960, Appendix D), and, at the same time, corresponds to the way in which von Neumann (1945) tackles this topic in his pioneering contribution. Henceforth, this representation of Fixed Capital in terms of joint production is referred to as the Torrens rule, since this latter author initially devised this method; as is quoted in Sraffa (1960, Appendix D).

The Torrens rule leads to device, for a production process that obtains a commodity, an equation system representing successive processes where machines at different stages of wear and tear are obtained and re-used by the same firm until they are scrapped. Since these machines are treated as differentiated commodities with their own price, the profit rate is calculated in each production process using the book value of fixed capital stock derived from the prices of these used machines.

However, one can devise an alternative method for representing Fixed Capital within a Sraffian framework introducing a slight, but significant, modification to the equations representing successive use of fixed capital stock within a firm. This leads to the profit rate in successive production processes being calculated using the value of fixed capital stock as new, instead of calculating the profit rate using the fixed capital stock book value at the beginning of each period, even though machines suffer wear and tear depreciation over time. The core of this paper is precisely to provide the rationale for this procedure that at first sight seems to be absurd.

The object of this new method for representing Fixed Capital is to get rid of the fixed capital stock book values associated with the price of machines at different stages of wear and tear; since these prices are in fact accounting values, not market prices, i.e. they do not regulate any transaction in an organized market. In this way, the new method is better adapted than the Torrens rule to prove that only market prices matter for performing the role of reproduction of economic system as a whole, as it occurs in a natural way in the context of Sraffian Circulating Capital models.

Notwithstanding this, one main conclusion reached in this paper is that it could be legitimate to combine the fixed capital stock valued as new with a proportional depreciation, as is widespread practice within input-output analysis.

This paper is structured as follows. The second section describes the Sraffian framework for the analysis. The third and fourth sections contain brief expositions, respectively, of the Torrens rule and the new method for representing Fixed Capital. The fifth section is devoted to the use of the so-called Replacement Fund for fixed capital. The sixth section contemplates a possible generalization of this new method for the case of constant efficiency of fixed capital stock. Finally, some concluding remarks are pointed out.
2. ANALYTICAL FRAMEWORK

The analysis of Fixed Capital is developed within the following Sraffian framework:

a) Absence of joint production. Thus, a technique as a whole consists of one production process to obtain each commodity separately. We consider $n \geq 2$ commodities\(^1\); hence, there are $n$ production processes.

b) Production processes are point/flow input-point output.

c) By definition, the duration of a production process is the time elapsed from when the first input is incorporated until the corresponding output is obtained. We consider that all production processes are of the same duration; and we take the latter as a unit of time.

d) Homogeneous labor is the sole primary input.

e) Wages are paid *ex post*, *i.e.* at the end of each production process. The time period for wage payment coincides then with the duration of production processes.

f) The profit rate is referred to the unit of time, as well as calculation of machines’ periodic allowance for depreciation.

g) Wage and profit rates are uniform throughout the whole economy.

Let us take, for instance, the $n$-th commodity within the technique as a whole that we are considering. As regards fixed capital goods used to produce this commodity the following assumptions hold for the sake of simplification:

h) Machines have physical constant efficiency throughout their economic lifetime.

i) All machines utilized have the same economic lifetime.

j) Scrapped machines are deprived of any residual value, *i.e.* there are no markets for such commodities. Otherwise, the system would give rise to joint production processes in strict sense, which are explicitly avoided.

Mathematical notation

a) Suppose that the first $h$ commodities ($1 \leq h < n$) are machines, which are used to produce the $n$-th commodity. The rest are circulating capital goods. The fixed capital stock used to produce this commodity has constant efficiency and all the machines have the same economic lifetime ($t \geq 2$). $t$ stands for the number of production cycles (the duration of each one as one unit of time) in which each machine is used.

b) $\kappa_n$ stands for a row vector with $n$ components, the first $h$ of which are positive and the rest are zero. It denotes the number of units of each new machine employed to produce the $n$-th commodity.

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\(^1\) With only one good there is no room for exchange between commodities, and so a Market Economy is actually ruled out.
c) When producing the $n$-th commodity we obtain an output of $b_n$ units in each period of time, employing a vector $a_n$ of material inputs and a quantity $l_n$ of direct labor, due to the constant efficiency of the fixed capital stock. $a_n$ stands for a row vector with $n$ components, the first $h$ of which are zero.

d) The wage rate is represented by $w$, and $r$ stands for the profit rate, referred to the unit of time that we have assumed above.

e) Let $p$ be a column vector of $n$ components. It stands for the price vector of marketed commodities. The first $h$ components correspond respectively to the price of new fixed capital goods, and the $n$-th component to the price of the $n$-th commodity.

3. THE TORRENS RULE

It is a well-known assumption that the $n$-th production process, from which we obtain the $n$-th commodity, unfolds in $t$ processes, which embrace successive use of fixed capital stock over time until this stock is completely replaced. In this way, it can be interpreted that these successive processes take place within a firm, say, operating over time.

Let $k^1_h, j = 1, \ldots, t$ be a row vector with $h$ components, which represents the number of the $h$ fixed capital goods in successive phases of wear and tear employed within the $n$-th production process. Such worn capital goods are considered as independent commodities, qualitatively differentiated from one another in this standard method for representing Fixed Capital in terms of joint production. In fact, they may be termed as intermediate capital goods. $k^1_h$ stands for machines depreciated at the end of the first period of time, and $k^t_h$ for the same set of depreciated fixed capital goods at the end of $t$-th period.

Let $p^1_h, j = 1, \ldots, t$ be a column vector of $h$ components, which represents the prices for the respective intermediate capital goods. These prices are in fact accounting values, not market prices. Intermediate capital goods are not actually demanded by other firms or by consumers; on the contrary, they are re-used in obtaining the same commodity (i.e. by the same firm) until they are scrapped and replaced by new machines.

The fixed capital stock book value at the end of successive periods can be expressed as follows:

$$VT_j = k^j_h p^j_h \quad j = 1, 2, \ldots, t \quad (1)$$

---

2 If these prices for intermediate capital goods were actually market prices, then each of the $t$ production processes can be identified with the activity of an independent firm. In that case, all machines one-year-older are to be replaced at the end of each production process by all firms, and then each firm begins a new production cycle. Therefore, these machines would in fact be circulating capital goods, i.e. there would not be any qualitative difference between $k_n, k^1_h, j = 1, \ldots, t$ and $a_n$. 

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By definition, allowance for depreciation in any one single period is equal to fixed capital stock book value at the beginning of that period minus its book value at the end; thus, allowance for depreciation can be obtained as follows:

\[ \Delta_j = VT_{j-1} - VT_j \quad j = 1, \ldots, t \quad VT_0 = k_n p \]  

(2)

Hence, it can easily be inferred that:

\[ \sum_{j=1}^{t} \Delta_j = VT_0 - VT_t = k_n p - VT_t \]

Then

\[ VT_t = k_n p - \sum_{j=1}^{t} \Delta_j \]

(3)

i.e. fixed capital stock book value at the end of \( i \)-th period equals the value of the stock when new less accumulated depreciation up to this date.

Since overall accumulated depreciation throughout \( t \) periods must be equal to the value of fixed capital stock when new:

\[ k_n p = \sum_{j=1}^{t} \Delta_j \]  

(4)

it turns out that \( VT_t = 0 \); consequently, it holds \( p_c^t = 0 \).

According to the Torrens rule, equations displaying successive use of fixed capital stock in obtaining the \( n \)-th commodity turn out to be as follows:

\[ \begin{align*}
\rho k_n p + k_n p + (1 + r) a_n p + w_n &= VT_0 + b_n p_n \\
\rho VT_0 + VT_1 + (1 + r) a_n p + w_n &= VT_2 + b_n p_n \\
\vdots \\
\rho VT_{t-2} + VT_{t-1} + (1 + r) a_n p + w_n &= VT_{t-1} + b_n p_n \\
\rho VT_{t-1} + VT_t + (1 + r) a_n p + w_n &= 0 + b_n p_n 
\end{align*} \]

(5)

Or, according to (2):

\[ \begin{align*}
\rho k_n p + \Delta_1 + (1 + r) a_n p + w_n &= b_n p_n \\
\rho VT_1 + \Delta_2 + (1 + r) a_n p + w_n &= b_n p_n \\
\vdots \\
\rho VT_{t-2} + \Delta_{t-1} + (1 + r) a_n p + w_n &= b_n p_n \\
\rho VT_{t-1} + \Delta_t + (1 + r) a_n p + w_n &= b_n p_n 
\end{align*} \]

(6)
As can be seen, this method for representing Fixed Capital calculates the profit rate from the book value of fixed capital stock at the beginning of each period of time, which is the same as that corresponding to the end of the preceding period.

4. AN ALTERNATIVE METHOD FOR REPRESENTING FIXED CAPITAL

The motivation behind this alternative method is to provide a framework that allows us to consider in the analysis different ways of financing fixed capital stock by capitalist firms, which seek profitability from the whole of assets displayed in their balance sheet at the beginning of each period of time. To this end, it is fundamental to understand the coherence of the successive balance sheets that will be presented in this section, referred, to begin with, to a self-financing firm.

Consider that a firm is managing the \( t \) processes corresponding to a successive utilization of fixed capital stock \( k_n \) until its complete replacement. At the beginning of first period, according to the first equation of system (6), the firm’s initial balance sheet turns out to be as follows:

<table>
<thead>
<tr>
<th>Assets</th>
<th>Equities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circulating Capital</td>
<td>( a_n p )</td>
</tr>
<tr>
<td>Fixed Capital</td>
<td>( k_n p )</td>
</tr>
<tr>
<td></td>
<td><strong>Owners’ Equity</strong></td>
</tr>
<tr>
<td></td>
<td>Contributed Capital</td>
</tr>
<tr>
<td></td>
<td>( k_n p + a_n p )</td>
</tr>
</tbody>
</table>

This balance sheet indicates that, at the beginning of the firm’s activity, the capital value to be advanced amounts to \( k_n p + a_n p \), for covering needs of circulating capital \( (a_n) \) for purchasing material inputs to obtain output, and for the need for a fixed capital stock \( (k_n) \).

At the end of first period, output \( b_n \) is obtained and sold in the market by the firm. Then, circulating capital value \( a_n p \), that was spent in the acquisition of material inputs at the beginning, is transformed into \( b_n p_n - w l_n \), i.e. the revenue obtained by selling output minus wage payments. Additionally, at the same time, the firm has obtained a gross profit amounting to \( b_n p_n - w l_n - a_n p \), which appears under the title of owners’ equity. Then, the firm’s resulting balance sheet is:

<table>
<thead>
<tr>
<th>Assets</th>
<th>Equities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circulating Capital</td>
<td>( b_n p_n - w l_n )</td>
</tr>
<tr>
<td>Fixed Capital</td>
<td>( k_n p )</td>
</tr>
<tr>
<td></td>
<td><strong>Owners’ Equity</strong></td>
</tr>
<tr>
<td></td>
<td>Contributed Capital</td>
</tr>
<tr>
<td></td>
<td>( k_n p + a_n p )</td>
</tr>
<tr>
<td></td>
<td>Gross Profit</td>
</tr>
<tr>
<td></td>
<td>( b_n p_n - w l_n - a_n p )</td>
</tr>
</tbody>
</table>

Now, the firm has to charge the depreciation expense \( \Delta_n \) corresponding to the fixed capital stock in order to obtain the net profit to be distributed. Then, we get that the firm’s balance sheet becomes:
Actually, accounting for the depreciation cost of fixed capital does not lead to any payment, i.e. no circulating capital reduction, but to the appearance of a new account named *Accumulated Depreciation*. This is considered in accounting as a “contra account”, or an “offset account”, which means that some of the assets displayed in the firm’s balance sheet are overvalued. It occurs precisely with fixed capital stock, whose value at the end of first period is not actually \( k_n p \), as appears in the left hand side of this balance sheet, but precisely this amount minus Accumulated Depreciation \( \Delta_1 \), as follows:

\[
VT_1 = k_n p - \Delta_1
\]

where \( VT_1 \) is the book value of fixed capital stock at this date.

The firm proceeds now to distribute net profit, which according to the first equation of system (6) amounts to:

\[
b_n P_n - w_l n - a_n p - \Lambda_1 = r (k_n p + a_n p)
\]

Then, the resulting balance sheet after net profit distribution turns out to be the following:

<table>
<thead>
<tr>
<th>Assets</th>
<th>Equities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circulating Capital ( b_n P_n - w_l n )</td>
<td>Accumulated Depreciation ( \Delta_1 )</td>
</tr>
<tr>
<td>Fixed Capital ( k_n p )</td>
<td>Owners’ Equity</td>
</tr>
<tr>
<td></td>
<td>Contributed Capital ( k_n p + a_n p )</td>
</tr>
</tbody>
</table>

Recalling once again the first equation of system (6), it is easily obtained that:

\[
b_n P_n - w_l n - r (k_n p + a_n p) = a_n p + \Delta_1
\]

Therefore, this final balance sheet can be rewritten to constitute the one corresponding to the beginning of second period:
As can be seen in this balance sheet, at the beginning of second period the firm has recovered circulating capital for purchasing material inputs $a_n p$, to begin the second production cycle, and, at the same time, an additional circulating capital is available for the firm, amounting to $\Delta_1$, for the replacement of the stock of fixed capital at the end of its economic lifetime. This additional circulating capital is in fact the share of gross profit that is retained because the firm has charged the depreciation expense for fixed capital. Thus, Accumulated Depreciation can be understood as the source of a fund for fixed capital stock replacement that appears as an asset in the firm’s balance sheet. Alternatively, this fund can be interpreted as the materialization or investment of Accumulated Depreciation.

The final balance sheet can be redrawn as follows:

<table>
<thead>
<tr>
<th>Assets</th>
<th>Equities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circulating Capital</td>
<td>Accumulated Depreciation $\Delta_1$</td>
</tr>
<tr>
<td>For Purchasing Material Inputs</td>
<td></td>
</tr>
<tr>
<td>$a_n p$</td>
<td></td>
</tr>
<tr>
<td>For Replacing Fixed Capital</td>
<td>Owners’ Equity</td>
</tr>
<tr>
<td>$\Delta_1$</td>
<td>Contributed Capital $k_n p + a_n p$</td>
</tr>
<tr>
<td>Fixed Capital</td>
<td></td>
</tr>
<tr>
<td>$k_n p$</td>
<td></td>
</tr>
<tr>
<td>Deduct:</td>
<td></td>
</tr>
<tr>
<td>Accumulated Depreciation $\Delta_1$</td>
<td></td>
</tr>
</tbody>
</table>

In this new method for representing fixed capital, the profit rate is always calculated from the total value of equities (i.e. total capital value tied to the firm) displayed in the corresponding balance sheet at the beginning of each period. In this way, it is necessary to implement this procedure into a new (or alternative) second equation for system (6), as follows:

### Notes

3. This corresponds to second equation of system (6).

4. In this particular case, total equities coincide with owners’ equity. However, in general, total equities are the lump sum of liabilities plus owners’ equity. Then, when the profit rate is calculated from total equities this method for representing Fixed Capital becomes independent of the firm’s financial structure.
\[ r (k_n p + a_n p) + \Delta_j + a_n p + \omega I_n = b_n p \] (8)

As is evident from this equation, this new method for representing Fixed Capital calculates the profit rate in the second period from the total capital value \( k_n p + a_n p \) displayed in the latter firm’s balance sheet (corresponding to the beginning of this period); since total capital value tied to a firm is the lump sum of total asset value displayed on the left hand side of the balance sheet. In this particular case, the profit rate is calculated from the lump sum of the following assets: \( i) \) circulating capital for purchasing material inputs \( a_n p \) to obtain output; \( ii) \) replacement fund \( \Delta \) for fixed capital; and \( iii) \) fixed capital stock book value at this date \( VT_1 \). This book value can be calculated from (7), after subtracting Accumulated Depreciation \( \Delta \) up to this date from the value of fixed capital stock as new \( k_n p \).

In contrast, if we applied the Torrens rule, we should calculate the profit rate in the second period\(^5\) from the lump sum of circulating capital value for purchasing material inputs \( a_n p \) to obtain output, plus fixed capital stock book value at this date \( VT_1 \). In this way, the Torrens rule does not consider the replacement fund \( \Delta \) for fixed capital as an asset tied to the firm, even though the depreciation expense has been actually charged by the latter. In other words, the Torrens rule would not calculate the profit rate from the total capital amount \( k_n p + a_n p \) displayed in the firm’s balance sheet at the beginning of the second period in the case of a self-financing firm. This fact in any case seems to be contradictory with the behavior of a capitalist firm, which seeks profitability on overall capital value, i.e. on the whole of assets displayed in its balance sheet.

The same phenomenon occurs in the rest of the periods. Thus, repeating the same procedure, the following firm’s balance sheet referred to the beginning of \( h + 1 \)-th period is obtained:

<table>
<thead>
<tr>
<th>Assets</th>
<th>Equities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circulating Capital</td>
<td>Owners’ Equity</td>
</tr>
<tr>
<td>For Purchasing Material Inputs</td>
<td>Contributed Capital</td>
</tr>
<tr>
<td>( a_n p )</td>
<td>( k_n p + a_n p )</td>
</tr>
<tr>
<td>Replacement Fund</td>
<td></td>
</tr>
<tr>
<td>( \sum_{j=1}^{h} \Delta_j )</td>
<td></td>
</tr>
<tr>
<td>Fixed Capital</td>
<td></td>
</tr>
<tr>
<td>Deduct:</td>
<td></td>
</tr>
<tr>
<td>Accumulated Depreciation</td>
<td></td>
</tr>
<tr>
<td>( \sum_{j=1}^{h} \Delta_j )</td>
<td></td>
</tr>
</tbody>
</table>

In this firm’s balance sheet it is essential to observe that fixed capital stock depreciation gives rise to an additional circulating capital, as we mentioned above for the first period, named Replacement Fund, which necessarily equals Accumulated Depreciation. In this way, it can be interpreted that fixed capital stock

\(^5\) See the second equation of systems (5) and (6).
capital stock depreciation is a process of gradual conversion of fixed into circulating capital, which gives rise to this Replacement Fund for fixed capital. This fund offsets the decreasing book value of fixed capital stock over time due to wear and tear depreciation, so that the capital value displayed in the firm’s successive balance sheets corresponding to each period remains unchanged after net profit distribution.

Once the end of \( t \)-th period has been reached, Accumulated Depreciation equals the value of fixed capital stock as new, so that equation (4) holds.

Then, according to (3), the book value of fixed capital stock at this date vanishes \((VT_t = 0)\), and the firm proceeds to apply the Replacement Fund for substituting scrapped machines for new ones. After this operation, we get the firm’s initial balance sheet, with which our exposition began.

Therefore, equation system (6) can be rewritten according to this new method for representing Fixed Capital, by writing down the corresponding equations (from second period onwards) akin to (8), as follows:

\[
\begin{align*}
    r k_n p + \Delta_1 + (1 + r) a_n p + w l_n &= b_n p_n \\
    r k_n p + \Delta_2 + (1 + r) a_n p + w l_n &= b_n p_n \\
    \vdots & \quad \vdots \\
    r k_n p + \Delta_{t-1} + (1 + r) a_n p + w l_n &= b_n p_n \\
    r k_n p + \Delta_t + (1 + r) a_n p + w l_n &= b_n p_n
\end{align*}
\]  

(9)

Given (2), this system is equivalent to the following:

\[
\begin{align*}
    r k_n p + k_n p + (1 + r) a_n p + w l_n &= VT_1 + b_n p_n \\
    r k_n p + VT_1 + (1 + r) a_p p + w l_n &= VT_2 + b_n p_n \\
    \vdots & \quad \vdots \\
    r k_n p + VT_{t-2} + (1 + r) a_n p + w l_n &= VT_{t-1} + b_n p_n \\
    r k_n p + VT_{t-1} + (1 + r) a_n p + w l_n &= 0 + b_n p_n
\end{align*}
\]  

(10)

Note that the first equation of systems (10) and (9) is the same as that corresponding to systems (5) and (6), respectively. The rest of equations differ\(^6\).

When considering (9) and (10), it is important to underline that at first sight this alternative method for representing Fixed Capital seems to be wrong, since the fixed capital stock is valued as new in all of the equations composing both systems, even though this stock is subject to wear and tear depreciation over time.

\(^6\) Naturally, the resulting prices for commodities, periodic allowance for depreciation and successive book values in systems (5) and (6) are normally different from those obtained in systems (9) and (10). We do not point out this fact in order to save notation.
However, it can be assured from the preceding argument that the amount of capital value tied to the firm (i.e. the value of total equities $k_n p + a_n p$) remains unchanged at the beginning of the $t$ periods, after net profit distribution. It is due to the counterbalance effect, disregarded by the Torrens rule, of the Replacement Fund in offsetting decreasing book values of fixed capital stock. In this way, from the firm’s final balance sheet, it holds that:

$$k_n p + a_n p = \sum_{j=1}^{n} \Delta_j + \sqrt{T_h} + a_n p \quad h = 1, \ldots, t$$

where first term of right hand side is the Replacement Fund.

This is the reason why this new method seems to be absurd at first sight, as long as it is apparently representing everlasting machines that at the same time suffer wear and tear depreciation. Rather to the contrary: The preceding argument, grounded on a firm’s financial accounting, clearly indicates the paradox of an unchanged capital value through successive balance sheets, despite fixed capital stock suffering wear and tear depreciation over time, i.e. corresponding book values are decreasing. This paradox underlies equation systems (9) and (10), which are the materialization at first instance of this new method for representing Fixed Capital.

5. THE REPLACEMENT FUND FOR FIXED CAPITAL

In the preceding section we have considered the case of a self-financing firm, where owners’ contributed capital coincides with total equities.

Let us now consider the case in which there are some liabilities in firm’s balance sheet, i.e. the firm has incurred in some debts when financing advanced capital. In this case, we have the following initial balance sheet:

<table>
<thead>
<tr>
<th>Assets</th>
<th>Equities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circulating Capital</td>
<td>$a_n p$</td>
</tr>
<tr>
<td>Fixed Capital</td>
<td>$k_n p$</td>
</tr>
<tr>
<td>Liabilities</td>
<td>$(1 - \alpha) ,(k_n p + a_n p)$</td>
</tr>
<tr>
<td>Owners’ Equity</td>
<td>Contributed Capital $\alpha(k_n p + a_n p)$</td>
</tr>
</tbody>
</table>

where $\alpha$ is the share of total capital displayed in the balance sheet (total equities) that is disbursed by the owners.

Net profit distributed at the end of first period corresponding to total equities is composed of the interest payments to capital borrowed (interest rate $r^*$) plus the profit corresponding to the owners’ contributed capital (profit rate $r^*$), as follows:

$$r(k_n p + a_n p) = \left[r^* \alpha + r^* (1 - \alpha)\right] (k_n p + a_n p) \quad r^* < r < r^{**}$$
As is usual, borrowed capital renders an interest rate to the lenders that is lower than the profit rate received by the owners, who hold property rights and control the firm assuming risk\(^7\).

If a part or all of the profit accruing to the owners in some period is used to pay back some of the borrowed principal, or the owners disburse an additional capital for the same purpose, we have the following balance sheet corresponding, for example, to the end of the \(h\)-th period:

<table>
<thead>
<tr>
<th>Assets</th>
<th>Equities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circulating Capital</td>
<td>Liabilities</td>
</tr>
<tr>
<td>For Purchasing</td>
<td>((1 - \alpha) (k_n p + a_n p) - (CE + RP))</td>
</tr>
<tr>
<td>Material Inputs</td>
<td></td>
</tr>
<tr>
<td>Replacement Fund</td>
<td></td>
</tr>
<tr>
<td>(\sum_{j=1}^{h} \Delta_j)</td>
<td></td>
</tr>
<tr>
<td>Fixed Capital</td>
<td>Owners' Equity</td>
</tr>
<tr>
<td>Deduct:</td>
<td>Contributed Capital</td>
</tr>
<tr>
<td>(k_n p)</td>
<td>(\alpha(k_n p + a_n p) + CE)</td>
</tr>
<tr>
<td>Accumulated Depreciation</td>
<td>Retained net profit</td>
</tr>
<tr>
<td>(\sum_{j=1}^{h} \Delta_j)</td>
<td>(RP)</td>
</tr>
</tbody>
</table>

where \(CE + RP\) stands for financial amortization of debt carried out up to the end of \(h\)-th period, and \(CE\) denotes enlargement of contributed capital by the owners referred to the same date.

As we can see, the situation does not become altered with respect to that described in the preceding section. Now the amount of liabilities decreases as owners’ equity increases, so that total equities remain unchanged. In other words, only the firm’s financial structure is altered with respect to the situation described in the preceding section.

In the context of constant efficiency of fixed capital stock, which we are considering, the Replacement Fund is positive in all periods. Let us suppose that in all periods the whole Replacement Fund is invested within the firm, for instance as inventories of circulating capital goods; this is in fact a limiting case, very rare in the context of constant efficiency of fixed capital stock.

We are dealing with a capitalist firm, which seeks profitability on the whole of assets displayed in its balance sheet at the beginning of each period. Since the new method is actually applying the profit rate \(r\) to the whole of assets, the firm is receiving a profit rate from the whole of its assets (i.e. a rate of return, hereinafter) uniform in all periods. By contrast, since the Torrens rule disregards the Replacement Fund, it is applying the profit rate \(r\) to part of the assets in each period; then, the firm’s rate of return decreases as the book value of fixed capital stock diminishes. Hence, the firm is actually receiving a rate of return that is not uniform in all periods, and it is lower than \(r\) in all periods except the first.

\(^7\) Despite \(r\) being assumed uniform for the whole economic system, \(\gamma^{**}\) is normally not, since it depends on firm’s financial structure, which varies from sector to sector. The operation carried out by a firm for enhancing profit rate \(\gamma^{**}\) by borrowing some capital at interest rate \(\gamma^{-}\) is known as financial leverage or debt leverage.
Let us consider the case in which the Replacement Fund is not applied to replace machines at the end of their economic lifetime, but rather to pay back some borrowed capital, that is, to reduce the amount of liabilities without altering owner's equity.

In this context, at the end of the \( h \)-th period we have the following balance sheet (as opposed to the previous one):

\[
\begin{array}{c|c}
\text{Assets} & \text{Equities} \\
\hline
\text{Circulating Capital} & \text{Liabilities} \\
\text{For Purchasing} & (1 - \alpha)(k_nP + a_nP) - \sum_{j=1}^{h} \Delta_j \\
\text{Material Inputs} & \alpha(k_nP + a_nP) \\
\text{Fixed Capital} & \text{Owners' Equity} \\
\text{Deduct:} & \\
\text{Accumulated Depreciation} & \sum_{j=1}^{h} \Delta_j \\
\end{array}
\]

As can be seen, the Replacement Fund has disappeared as an asset tied to the firm, since it has been used at the end of each period to pay back to the creditors some of the borrowed capital. In this case, the total capital tied to the firm decreases as the fixed capital stock depreciates over time, \( i.e. \) as the successive book values of fixed capital stock get lower. And so, the machines’ replacement at the end of their economic lifetime is actually carried out without resorting to any Replacement Fund, but by borrowing new capital from an outside financial source, for purchasing new machines to substitute the scrapped ones.

In this case the Torrens rule would be correctly calculating the profit rate from the whole of assets displayed in firm’s balance sheet, as can be seen by direct inspection of the last balance sheet. Then, the rate of return \( r \) received by the firm turns out to be uniform in all periods when one applies the Torrens rule. By contrast, the new method for representing fixed capital is applying the profit rate \( r \) in each period to a constant and fictitious capital value, which is larger than the real one displayed in the latter balance sheet. Thus, the rate of return received by the firm is actually higher than \( r \) from the second period onwards, and it is increasing as the book values decrease. Therefore, the rate of return received by the firm is not uniform in all periods, the same as in the preceding case for the Torrens rule.

It should be underscored that to make the Replacement Fund disappear in each period as an asset belonging to the firm’s balance sheet, we require that the two following conditions hold:

\( a) \) The whole fixed capital stock is to be financed by borrowing capital (leasing contracts, bond issues, loans, etc.).

\( b) \) The stream of financial payments of principal to the lenders must exactly coincide with the stream of depreciation quotas of fixed capital stock during its economic lifetime.
It is clearly difficult that both conditions always hold, since the depreciation quota of fixed capital stock may become negative in some periods, outside of the simplified case of constant efficiency. In addition, only in this latter case does the annual charge for fixed capital (i.e. depreciation plus interest payment) become constant throughout all periods. So it is difficult, or even impossible, in the case of variable efficiency, to match the financing of fixed capital to an installment plan that depends on the pattern of machines’ depreciation profile over time. Hence, this case is to be considered as a limiting case, a counterpart of the preceding one, even though it can be considered normal in the context of constant efficiency of fixed capital stock.

Outside of this limiting case in which the Replacement Fund completely disappears as an asset belonging to firm’s balance sheet, if part of this fund has been used in some periods to pay back some borrowed capital, the Replacement Fund appears as an asset but is smaller than Accumulated Depreciation. And in this mixed case neither method for representing fixed capital would be calculating the profit rate from the overall asset value displayed in firm’s balance sheet.

In fact, the new method overestimates the capital value displayed in balance sheet in all periods when calculating the profit rate, and the Torrens rule underestimates it. Thus, the resulting rate of return received by the firm, calculated from the overall asset value displayed in its balance sheet at the beginning of each period, turns out to be lower than \( r \) for the Torrens rule and higher than \( r \) for the new method; and in both cases it is normally not uniform in all periods.

Finally, let us consider again the case contemplated at the beginning of this section, where the Replacement Fund for fixed capital is actually used to replace machines at the end of their economic lifetime and not to reduce liabilities. In this case, the amount of Replacement Fund does coincide with the amount of Accumulated Depreciation, so that the new method is correctly calculating the profit rate from the whole of assets displayed in the firm’s balance sheet (the second one of this section).

The question now is how and where to invest this Replacement Fund in order to render some additional revenue to that obtained from selling output, which is indeed the firm’s main activity.

Part of the Replacement Fund can be invested inside the firm, for instance as stocks of circulating capital goods; and the rest can be lent to other firms or to consumers through the financial market, by buying up some liquid financial assets\(^8\).

Let us take, for instance, the \( h \)-th equation of system (9), referred to the \( h \)-th period. We can write:

\[
x (k_n p + a_n p) + x_h \beta_h \sum_{j=1}^{h-1} \Delta_j + \Delta_h + a_n p + w l_n = b_n p_n + x_h' \beta_h \sum_{j=1}^{h-1} \Delta_j \quad h = 2, \ldots, \tau
\]

\(^8\) Financial assets to invest the Replacement Fund are to be liquid enough for being rescued in due course either to replace scrapped machines as well as to offset negative depreciation quotas in some periods corresponding to fixed capital stock in the context of variable efficiency. And, as is a well-known fact, liquidity means a lower yield rate than other less liquid financial assets.
where $r^*_h$ stands for the interest rate in the $h$-th period yielded by the firm’s financial portfolio, as investment outside of a share $0 \leq \beta_h \leq 1$ of the replacement fund corresponding to the end of the $h-1$-th period.

$r(k_nP + a_nP)$ is the firm’s operating net profit obtained from selling output, which is the firm’s main activity. In fact, we have added to both sides of the $h$-th equation of (9) the financial revenue obtained in the $h$-th period corresponding to the firm’s financial portfolio.

Hence, we can express the firm’s total profits obtained in the $h$-th period by a profit rate calculated from the whole of assets displayed in the firm’s balance sheet, i.e. $r^*_h$ is now the firm’s rate of return. We get the following equation:

$$r^*_h \left( k_nP + a_nP \right) + \Delta_n + \omega l_n = b_nP_n + x^* \beta_h \sum_{j=1}^{h-1} \Delta_j$$

$$h = 2, \ldots, t$$

Comparing the two latter equations one obtains:

$$r^*_h = r + x^* \frac{\beta_h \sum_{j=1}^{h-1} \Delta_j}{k_nP + a_nP}$$

From this expression, it turns out that $r^*_h \geq r$, provided that $r^*_h \geq 0$ and $\sum_{j=1}^{h-1} \Delta_j > 0$, as we are assuming. In other words, the rate of return $r^*_h$ in the $h$-th period is always larger than the operating profit rate $r$, as long as the financial yield rate $r^*_h$ corresponding to financial assets is positive, and the firm holds some investment outside ($\beta_h > 0$). In the case in which the whole Replacement Fund is invested inside the firm ($\beta_h = 0$), we get $r^*_h = r$.

Finally, it is to be underscored that despite $r$ being uniform in all periods and for the whole economic system, $r^*_h$ is not; since the latter depends on the size of financial revenue obtained by the firm in each period.

Let us consider now the Torrens rule referred to the second balance sheet of this section, or to the last one of the preceding section. The Replacement Fund appears as an asset and coincides with Accumulated Depreciation.

Taking the $h$-th equation of system (6), we can write:

$$r \left( V_{n-1} + a_nP \right) + x^*_h \beta_h \sum_{j=1}^{h-1} \Delta_j + \Delta_n + a_nP + \omega l_n = b_nP_n + x^*_h \beta_h \sum_{j=1}^{h-1} \Delta_j$$

$$h = 2, \ldots, t$$

The firm obtains some financial revenue from holding a financial portfolio, where a share $\beta_n$ of the Replacement Fund has been invested; the rest has been invested within the firm.

---

9 In fact, net profit defined in this way includes interest payment to capital borrowed by the firm.
We can express the total profits obtained by the firm by means of a profit rate \( r_{h}^{*} \) calculated from the whole of assets (total equities), i.e. \( r_{h}^{*} \) is now the firm’s rate of return. Doing so yields the following equation:

\[
r_{h}^{*} \left( k_{h}p + a_{h}p \right) + \Delta_{h} + a_{h}p + \omega l_{n} = \delta_{h}p_{n} + r_{h}^{*} \beta_{h} \sum_{j=1}^{h-1} \Delta_{j} \quad h = 2, \ldots, t
\]

Comparing these last two equations we obtain:

\[
r_{h}^{*} \left( k_{h}p + a_{h}p \right) = r \left( VT_{h-1} + a_{h}p \right) + r_{h}^{*} \beta_{h} \sum_{j=1}^{h-1} \Delta_{j}
\]

Then, it holds that:

\[
r_{h}^{*} \left( k_{h}p + a_{h}p \right) = r \left( VT_{h-1} + \sum_{j=1}^{h-1} \Delta_{j} + a_{h}p \right) + r_{h}^{*} \beta_{h} \sum_{j=1}^{h-1} \Delta_{j} - r \sum_{j=1}^{h-1} \Delta_{j}
\]

Now, recalling (3), it turns out that:

\[
r_{h}^{*} = r - \left( r - r_{h}^{*} \beta_{h} \right) \frac{\sum_{j=1}^{h-1} \Delta_{j}}{k_{h}p + a_{h}p}
\]

where, according to (4), it holds that

\[
0 < \frac{\sum_{j=1}^{h-1} \Delta_{j}}{k_{h}p + a_{h}p} < 1
\]

If the whole Replacement Fund is invested inside the firm (\( \beta_{h} = 0 \)), it turns out that \( 0 < r_{h}^{*} < r \). Since the Torrens rule does not calculate operating profit rate \( r \) from the whole of firm’s assets, the rate of return \( r_{h}^{*} \) obtained by the firm from the whole of assets is smaller than \( r \). The latter value can be thought of as the rate of return obtained in the economic system by a firm that only employs circulating capital.

If the whole Replacement Fund is invested outside the firm (\( \beta_{h} = 1 \)), since it normally holds that \( r_{h}^{*} < r \), it turns out that \( r_{h}^{*} \) is smaller than \( r \), the same as above.

Then, in order that \( r_{h}^{*} = r \) holds we require that \( r_{h}^{*} = r \) and \( \beta_{h} = 1 \) both hold, i.e. the interest rate yielded by firm’s financial portfolio must be equal to firm’s operating profit rate and the whole Replacement Fund must be invested outside the firm\(^{10} \). Both conditions are unlikely to be fulfilled in all periods.

\(^{10}\) This proposition can be deduced directly from the \( h \)-th equation of (6), by summing financial revenue \( r \sum_{j=1}^{h-1} \Delta_{j} \) to both sides. This yields:
The rate of return $r_{tn}$ corresponding to the Torrens rule, calculated from firm’s total assets, is the counterpart of $r_{tn}^D$, the one obtained from the new method. Both rates of return may vary from period to period and are not necessarily uniform for the whole economic system, in contrast to the operating profit rate $r$.

Summing up. When the firm’s rate of return is required to be uniform over all periods, from the preceding argument it becomes clear that the Torrens rule is dealing with two limiting cases: when the replacement fund is positive in all periods i) it is used entirely in each period to reduce the firm’s liabilities incurred when financing the fixed capital stock with total debt; or ii) the replacement fund is invested in each period outside the firm by purchasing some financial assets, which have to render in each period a yield rate equal to firm’s operating profit rate $r$. From the argument contained in this section, this latter limiting case can be considered as unrealistic.

Outside of these two limiting cases, the Torrens rule does not guarantee that a firm using fixed capital receives in each period a rate of return $r_{tn}$ from total assets displayed in its balance sheet that is at least as large as that corresponding to a circulating capital industry, and so the operating profit rate $r$ cannot be uniform over the whole economic system in the context of free competition.

On the other hand, the new method allows the operating profit rate $r$ to be uniform in the whole economic system, since it guarantees in any case that a fixed capital industry is rewarded in each period with a rate of return $r_{tn}^D$ from total assets that is not smaller than $r$, the one corresponding to a circulating capital industry. However, a firm employing fixed capital normally receives a rate of return that is higher than $r$ the larger is the financial revenue yielded by its financial portfolio, so that the rate of return is not actually uniform in the new method throughout all periods and for the whole economic system. Except in one limiting case, which can be thought of as unrealistic in the context of constant efficiency: when the whole replacement fund, positive in all periods, is invested in each period inside the firm as inventories of circulating capital goods. In this case, contemplated at the beginning of this section, the firm’s rate of return is uniform over all periods, and equal to operating profit rate $r$.

The conclusion is evident. Irrespective of the method used for representing fixed capital, a uniform rate of return received by a firm in all periods, from total assets displayed in its balance sheet, is the exception; the rule is non-uniformity, since we are using a uniform operating profit rate $r$ in all periods.

Hence, at this stage of the argument it is indifferent which method to use for representing Fixed Capital, since both of them seem to be legitimate for calculating a uniform operating profit rate $r$ in all periods from the whole or part of the assets displayed in firm’s balance sheet.

Therefore, from the preceding argument we can conclude that neither of the two methods is preferred to the other. Both methods are in fact alternative and legitimate representations of Fixed Capital.

\[ r \left( k_n p + a_n p \right) + \Delta_n + a_n p + \omega I_n = b_n p + n + r \sum_{j=1}^{n-1} \Delta_j \quad h = 2, \ldots, t \]
to be considered in the same light; since only in limiting cases, which are needless to say unrealistic, each method is able to warrant the rate of return received by a firm from total assets being uniform in all periods and for the whole economic system in the context of free competition.

6. GENERALIZATION OF THE NEW METHOD

In this section we will generalize systems (9) and (10) to encompass any firm’s financial policy concerning the use of the replacement fund as discussed in the preceding section, and at the same time to warrant the firm’s rate of return being uniform over all periods.

As we have seen in the preceding section, the replacement fund for fixed capital can be lower than accumulated depreciation at any single time period, due to the firm’s financial policy of using part or the whole referred fund to reduce liabilities. Hence, the firm’s total asset value at the beginning of j-th period can in general be expressed as follows:

$$\theta_j k_n p + a_n p$$

where $0 < \theta_j \leq 1 \quad j = 1, \ldots, t$ in the context of constant efficiency of fixed capital stock, which we are considering.

In case that the whole replacement fund is used in each period to reduce firm’s liabilities, as the Torrens rule implicitly assumes, these parameters are lower than 1, except the first one, and monotonically decreasing, as it happens with the successive book values of fixed capital stock in (5) and (6), whose behavior these parameters are trying to imitate in this case$^{11}$.

Therefore, parameter $\theta_j$, referred to j-th period, is smaller than 1 if part of the firm’s replacement fund corresponding to preceding periods has been used to pay back to the creditors some of the borrowed principal. Obviously, it holds that $\theta_1 = 1$.

At the same time, the whole or part of the firm’s replacement fund in each period can be invested in the financial market for rendering some financial revenue, to complement the revenue obtained from selling the firm’s output. Hence, the firm’s rate of return from total assets can hardly be uniform over all periods as long as the operating profit rate $r$ is uniform, as we concluded in the preceding section.

Since we are interested in the firm’s rate of return being uniform over all periods, we have to apply a correction factor in each period to the uniform operating profit rate $r$, as follows:

$$r \rho_j \left(\theta_j k_n p + a_n p\right)$$

where $\rho_j > 0 \quad j = 1, \ldots, t$.

$\rho_j$ is a parameter smaller than 1 if the firm is receiving some positive financial revenue in the j-th period. Otherwise, this parameter becomes higher than 1 if the referred financial revenue is negative.

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$^{11}$ See Appendix for a formal proof.
Obviously, it holds that $\rho_1 = 1$, since the firm is not receiving any financial revenue in the first period.

After these considerations, the resulting equation system, as a generalization of (9), adopts the following shape:

$$
\begin{align*}
    r \rho_1 (\theta k_n p + a_n p) + \Delta_1 + a_n p + w I_n &= b_n p_n \\
    r \rho_2 (\theta k_n p + a_n p) + \Delta_2 + a_n p + w I_n &= b_n p_n \\
    \vdots & \vdots \\
    r \rho_t (\theta k_n p + a_n p) + \Delta_t + a_n p + w I_n &= b_n p_n
\end{align*}
$$

(11)

In the same way, a similar generalization of (10) can be obtained:

$$
\begin{align*}
    r \rho_1 (\theta k_n + a_n) p + k_n p + a_n p + w I_n &= VT_1 + b_n p_n \\
    r \rho_2 (\theta k_n + a_n) p + VT_1 + a_n p + w I_n &= VT_2 + b_n p_n \\
    \vdots & \vdots \\
    r \rho_t (\theta k_n + a_n) p + VT_{t-1} + a_n p + w I_n &= 0 + b_n p_n
\end{align*}
$$

(12)

In this order of things, system (11) can be easily particularized to replicate (6), corresponding to the Torrens rule, by choosing conveniently parameters $\theta_j$, $j = 2, \ldots, t$ smaller than 1 and monotonically decreasing, and at the same time by making that $\rho_j = 1$, $j = 1, \ldots, t^{12}$.

On the other hand, for the same purpose of warranting a uniform rate of return over all periods, the Torrens rule would have to introduce similar parameters $\theta$ and $\rho$ to contemplate all uses of the replacement fund by the firm, and so, to become a general method to represent Fixed Capital. If not, this standard method is confined to dealing with some limiting cases where the firm is actually receiving a uniform rate of return over all periods.

Therefore, we have reached the same conclusion as that obtained at the end of the preceding section, that is, at this stage of the argument it is indifferent which method to use for representing Fixed Capital. Hence, both methods are to be analyzed in the light of Sraffa’s theory of prices, to decide which one of them is more suitable for performing the reproduction of economic system, which is the main concern of this theory.

7. CONCLUSION

According to the argument developed in this paper, it can be claimed that the new method for representing Fixed Capital is as legitimate as the Torrens rule, the standard method commonly accepted within the Sraffian literature.

\[^{12}\] See Appendix.
This new method seems to be well grounded on elementary knowledge about Financial Accounting\textsuperscript{13}, which is not required for the Torrens rule.

This latter method, as long as it calculates the profit rate from the book value of the stock of fixed capital in each period, is forced to consider worn machines at different stages of wear and tear as differentiated commodities (each one with its own price), which gives rise to joint production processes (system (5)). On the other hand, the new method for representing Fixed Capital does not require that a worn machine be priced, whereupon joint production processes never arise.

This latter fact is suggested from system (11). Since all the equations must be simultaneously satisfied, any linear combination automatically holds; for instance, the one resulting from simple summation of the relevant equations. Then, recalling (4), we would have:

\[
x \left( k_n \sum_{j=1}^{t} \frac{\rho_j}{\tau} + a_n \sum_{j=1}^{t} \frac{\rho_j}{\tau} \right) p + \frac{1}{\tau} k_n p + a_n p + w_{1,n} = b_n p_0
\]

It suffices to introduce the latter as the \( n \)-th equation of the price system associated with the technique as a whole for producing the \( n \) commodities, and so the prices of marketed commodities and the wage rate can be determined for a given \( r \), and for any given positive parameters \( \theta_j \) and \( \rho_j \) \( j = 1, \ldots, t \). After this, allowance for depreciation and successive book values of fixed capital stock can be ascertained by using, respectively, systems (11) and (12).

This latter equation shows that this new method for representing fixed capital is reminiscent of the widely used practice within the input-output analysis of combining the fixed capital stock valued as new with a proportional depreciation. In this sense, we have provided the accounting foundation to this widespread practice, in the simplified context of constant efficiency of fixed capital stock\textsuperscript{14}.

In section four a very simplified introduction to this new method has been given for the case of constant efficiency of fixed capital stock and for machines of the same economic lifetime. This elementary exposition is justified for enhancing the distinctive features of this new method with respect to the Torrens rule.

However, this new method can be easily applied in a more general setup\textsuperscript{15}, where machines: \( i \) have variable efficiency; \( ii \) have different economic lifetime; and \( iii \) scrapped machines have a positive residual value (positive market price), i.e. they are actually demanded by other firms or by consumers in markets for second-hand fixed capital goods.

Finally, the question of which of the two methods is more suitable for the purpose of Sraffa’s Theory of Prices, centered in economic system reproduction, is left open for discussion. Still, from the


\textsuperscript{14} Nevertheless, the same accounting foundation is provided in a subsequent paper in the more general context of variable efficiency. See Ibanez, Matilla and Osuna (2004a).

\textsuperscript{15} See Ibanez, Matilla and Osuna (2004b).
preceding paragraphs one can guess that using the new method it would be straightforward to show that only market prices matter for performing the role of economic system reproduction, since this new method deals primarily with depreciation rather than with book values of fixed capital stock to determine market prices.
THE ANNUITY FORMULA OF THE TEXTBOOKS

It is a well-known fact in the context of fixed capital stock constant efficiency that the annuity formula appearing in the textbooks represents the annual charge for machines (depreciation plus interest payment), which turns out to be constant over all periods.

In effect, system (6) can be rewritten as follows:

\[ rVT_{j-1} + \Delta_j = Y(r) = b_jp_s - (1 + r)a_jp - wI_{n_j} \quad VT_0 \equiv k_n p \quad j = 1, \ldots, t \]

where \( Y(r) \) denotes the annual charge for machines, which obviously is constant over all periods.

The annuity formula results from (5) after multiplying each equation respectively by \((1 + r)^{t-j}\) \(j = 1, \ldots, t\), and then by summing up all the equations. Hence, preceding equation system can be rewritten as follows:

\[ \frac{r(1 + r)^{i}}{(1 + r)^{i} - 1} k_n p = rVT_{j-1} + \Delta_j = Y(r) \quad VT_0 \equiv k_n p \quad j = 1, \ldots, t \]

From the first equation of this system, allowance for depreciation turns out to be positive in the first period; and hence, it holds that \( k_n p > VT_0 \) according to (3).

Now, repeating the same procedure with the rest of preceding equations, it turns out that allowance for depreciation is positive and increasing over all periods, and so, the successive book values of fixed capital stock are positive and monotonically decreasing, until the last one vanishes.

The preceding analysis is valid not only in the case of physical constant efficiency that we are contemplating, but also in the more general case of constant efficiency in price terms, as follows:

\[ Y(r) = b_jp_s - (1 + r)a_jp - wI_{n_j} \quad j = 1, \ldots, t \]

i.e. annual charge for machines \( Y(r) \) is constant over all periods for a given value of profit rate, despite the output level and consumed inputs (material inputs and labor) may vary from period to period.

Now, proceeding in the same way as earlier from a system similar to (5), one obtains the annuity formula displayed above.

In the same way as in the standard method for representing fixed capital, the annuity formula is accurate for a representation of the annual charge for machines whenever the fixed capital stock is wholly financed with total debt, and all depreciation quotas corresponding to this stock are exclusively used to pay the principal back to the lenders.

Constant efficiency either in physical or in price terms is an unrealistic case of fixed capital stock efficiency. Notwithstanding this, it is to be conceded that the referred annuity formula normally applies in this odd case, since in this context it is advisable the use of an external source to finance the fixed capital stock by a firm.
This is so, because the annual charge for machines (depreciation plus interest payment) is constant over all periods in this particular case, and periodic allowance for depreciation is always positive. So, one can easily devise an installment plan to buy the whole fixed capital stock, by using periodic depreciation quotas corresponding to this stock to pay the principal back to the creditors.

This procedure is always possible, since in case of constant efficiency, the stocks for material inputs, which are sometimes required in the first period, are not required to be increased in subsequent periods, since the same production process repeats over all of them. And so, it is not necessary that the replacement fund for fixed capital is used as a firm’s internal financial source for creating inventories. Nor is this fund profitable when invested outside the firm in the financial market, since liquid financial assets, as advisable candidates for this investment, normally render a lower yield rate than the firm’s operating profit rate \( r \).

Therefore, in the unrealistic case of constant efficiency the following become normal practice: \( i \) the financing of fixed capital stock with total debt; and \( ii \) the exclusive use of the replacement fund as internal financial source to reduce firm’s liabilities incurred when financing this stock. And so, the standard method for representing Fixed Capital fully applies in this case, whenever all machines have the same economic lifetime.

Given this, this paper can be interpreted as an attempt to contemplate all possible forms to finance the fixed capital stock, and so, any firm’s financial structure, and any financial policy adopted by the firm about the use of the replacement fund for fixed capital as an internal financial source. The violation of the conditions required in the preceding paragraph does affect the depreciation pattern of fixed capital stock in such a way as to make the annuity formula inaccurate to represent the annual charge for machines in the context of constant efficiency.

In effect, if we take system (12) instead of (5), and we proceed as earlier, by multiplying each equation by \( (1 + r)^{t-j} \), \( j = 1, \ldots, t \), and then by summing up all equations, we do not get the annuity formula of the textbooks to represent the annual charge for machines, unless the following condition is fulfilled:

\[
\rho_j = 1 \quad \theta_j k_n p = v t_{j-1} \quad v t_0 = k_n p \quad j = 1, \ldots, t
\]

This condition makes systems (5) and (12) equivalent each other.

On the other hand, straight-line depreciation turns out to be an extremely unrealistic case, since all parameters \( \theta \) and \( \rho \) appearing in (11) are to be equal to 1; and so all the equations of (11) become equivalent.

Exactly this occurs when the whole replacement fund is invested in all periods inside or outside the firm \((\theta_j = 1 \quad j = 1, \ldots, t)\), and the latter is not receiving any financial revenue or we are disregarding it over all periods \((\rho_j = 1 \quad j = 1, \ldots, t)\).

In this latter more likely case, the firm is actually obtaining an increasing rate of return along the \( t \) periods, since we are calculating the operating profit rate \( r \) from the whole of assets displayed in firm’s balance sheet at the beginning of each period.
To conclude, this paper can be considered as an introduction to the accounting foundation of the new method for representing Fixed Capital, since this accounting foundation can be more easily carried out in the simplified context of physical constant efficiency, dealt with in this paper, than in the more general context of variable efficiency. In this sense, the introduction to the new method contained in this paper will be extended to this latter context in a subsequent paper.
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