Intertemporal substitution and the liquidity effect in a sticky price model

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Abstract

The liquidity effect, defined as a decrease in nominal interest rates in response to a monetary expansion, is a major stylized fact of the business cycle. This paper first confirms that, with separable preferences, a low degree of intertemporal substitution in consumption is a necessary condition for the existence of the liquidity effect. In contrast to this result, in a model with non-separable preferences and capital accumulation it takes an implausibly high elasticity of intertemporal substitution to produce a liquidity effect. The robustness of these results to alternative degrees of nominal rigidities, capital adjustment costs and stochastic monetary processes is also analysed. We conclude that price stickiness, by itself, does not guarantee the existence of a liquidity effect. © 2002 Elsevier Science B.V. All rights reserved.

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1. Introduction

The negative correlation between money growth and the nominal interest rate is one of the most salient features of the monetary transmission mechanism. Most researchers seek to identify positive monetary policy shocks as
those innovations to money growth that exert a positive influence on output (output effect) and prices (price effect), and that reduce the nominal interest rate on impact (liquidity effect). In fact, the confidence of the profession in this scheme is such that failure to produce any of these effects is taken as a puzzle that calls into question the identification procedure. Our reading of this literature is that the liquidity effect is a nominal feature that any well-defined monetary model of the business cycle must be able to produce under fairly general circumstances (see Christiano et al., 1999).

Two different types of frictions aim to account for the aforementioned effects, emphasizing different parts of the monetary policy transmission mechanism. On the one hand, limited participation models generate a liquidity effect by allowing restrictions on the adjustment of agents' portfolios which break down the intertemporal allocation of consumption (see Fuerst, 1992; Christiano et al., 1997). On the other hand, an alternative line of research has pointed to the role of price rigidities and capital adjustment costs as the main factors determining the behaviour of nominal and real interest rates after a money supply shock (see, for instance, King and Watson, 1996). The liquidity effect is not guaranteed in this class of models since it depends on how the nominal interest rate is affected by the intertemporal allocation of wealth.

Previous studies have analysed the liquidity issue in models where money growth followed an i.i.d. process (Dow, 1995) or capital accumulation was absent (Christiano et al., 1997). Moreover, those studies simply assumed that prices were predetermined one period in advance. We revisit the liquidity effect in a general equilibrium model in which agents face adjustment costs when changing both prices and capital, and where money services provide utility to consumers. This price setting is observationally equivalent to the one in Calvo's model (see Sbordone, 1998). As noted by Woodford (1996), this specification generates a Phillips trade-off where future inflation expectations play a crucial role in the paths of output and inflation. The price setting and the money process are key determinants of inflation expectations and so of the existence of a well-defined liquidity effect. In particular, we look for a negative response of nominal interest rates in the presence of persistent money growth shocks. Whereas that seems to be a good time series representation for the exogenous evolution of some monetary aggregates, we will also analyse how the existence of a liquidity effect depends upon the degree of persistence of the monetary process.

We confirm in our model that when preferences are separable, a positive monetary shock induces a fall in interest rates if the degree of intertemporal substitution is low enough (i.e. the risk aversion parameter is high enough) to generate a large impact response of current consumption relative to future consumption. This result has been noted in, inter alia, Jeanne (1994) and Christiano et al. (1997).
The previous result is obtained with separable preferences between consumption and leisure. Although separable preferences have been used in many monetary models, they can hardly be reconciled with the balanced growth properties unless some parametric restrictions are satisfied (see Chari et al., 1998). In a model with non-separable preferences, things are different since, after a monetary shock, consumption and labour movements induce counter-acting effects on the marginal utility of consumption. Furthermore, we will show that if there is no capital accumulation, the model never produces a liquidity effect, at least within the range of reasonable risk aversion values. This is so when preferences assume no income effect in labour supply, but also in the more general case of Cobb–Douglas preferences.

In order to obtain a liquidity effect, we must allow for capital accumulation in the economy. In that case, the fall in real interest rates allows households to postpone consumption and to allocate the increase in output to investment. With Cobb–Douglas preferences, a very low risk aversion parameter is a necessary condition to generate a fall in the nominal interest rate following a monetary shock. This is most unfortunate since it generates implausibly large impact responses in output, employment and investment. The impact response of the nominal interest rate also depends on other parameters of the model like the degree of nominal (price) and real (capital adjustment costs) inertia. Nevertheless in our model, these frictions are of secondary importance when compared with preferences, as regards the ability of the model to generate the liquidity effect. Thus, we conclude that accounting for the observed liquidity effect in sticky price models is not straightforward and that additional restrictions on the preferences (e.g. habit formation) and/or alternative specifications of the monetary policy may be required for that purpose.

The rest of the paper is organized as follows. Section 2 presents the model and defines the equilibrium. In Section 3, the model is calibrated to be compatible with a well-behaved steady state. Section 4 contains the main results of the paper. Section 5 concludes with some additional remarks.

2. The model

Households (indexed by $i$) maximize their expected lifetime utility $\tilde{U}_{i0}$, defined as the present discounted value of the momentary utility $U_{it}$ conditional on the information available at $t = 0$. They choose a joint plan for consumption ($C$), leisure ($1 - L$) and end-of-period real balances ($M/P$), where the utility of real balances stems from the transaction services provided by money:

$$\tilde{U}_{i0} = E_0 \left[ \sum_{t=0}^{\infty} \beta^t U_{it} \left( C_{it}, \frac{M_{it}}{P_t}, L_{it} \right) \right].$$  (1)
Each household accumulates capital and rents it to firms. The accumulation of capital is driven by

\[ K_{it} = I_{it-1} + (1 - \delta)K_{it-1}, \tag{2} \]

where \( \delta \) is the rate of depreciation, \( K \) is capital and \( I \) is investment. Adjusting capital to its desired level is costly for the households who own it; the function of adjustment costs is chosen to produce non-zero costs in the steady state:\(^1\)

\[ AC_{it}^K = \frac{\phi_k}{2} \left( \frac{I_{it}}{K_{it}} \right)^2 I_{it}, \tag{3} \]

where \( \phi_k \) is the adjustment cost parameter. Households hold their non-human wealth in the form of money (\( M \)), bonds (\( B \)) or capital, and earn labour income (\( PWL \)), income from their share (\( \omega \)) of profits (\( \pi \)), income derived from renting capital (\( PZK \)), interest payments from bonds and transfers (\( PT \)) from the government. The gross nominal interest rate is defined as \( R_t \). The budget constraint faced by each household can be written as follows:

\[ P_t C_{it} + M_{it} + B_{it} + P_t I_{it} \left( I + \frac{\phi_k}{2} \left( \frac{I_{it}}{K_{it}} \right)^2 \right) = P_t T_{it} + P_t W_{it} L_{it} + P_t Z_t K_{it} + M_{it-1} + R_{t-1} B_{it-1} + P_t \sum_{j=1}^{J} \omega_{ij} \pi_{jt}. \tag{4} \]

There are \( J \) firms indexed by \( j \). An aggregator transforms heterogeneous goods (\( Y_j \)) into a composite good (\( Y \)). More formally, the problem faced by the aggregator can be stated as follows:

\[ \text{Max}_{\{Y_j\}} PY - \sum_{j=1}^{J} P_j Y_j, \quad \text{where } Y = J^{1/\theta} \left[ \sum_{j=1}^{J} Y_j^{(\theta-1)/\theta} \right]^{\theta/(\theta-1)}, \]

where \( \theta \) is the elasticity of substitution between different goods and \( P_{jt} \) and \( P_t \) are the individual and aggregate output prices, respectively. Since the elasticity of substitution between the different goods is finite and greater than 1, each firm has some monopoly power and cares about its own price relative to the aggregate one. The first order conditions of this problem with respect to \( Y_j \) yield the

\(^1\) These real adjustment costs are paid through the purchase of a CES basket of all the produced goods of the economy (see Hairault and Portier, 1993).
following inverse demand schedule:\(^2\)

\[ P_{jt} = P_t \left( \frac{J Y_{jt}}{Y_t} \right)^{-1/\theta}. \]  

(5)

Marginal costs are increasing in output. With flexible prices, these functional forms imply that, in the steady state, firms charge a price-markup over marginal cost equal to \(\theta/(\theta - 1)\). This monopolistic competition environment makes it possible to incorporate sticky prices into the model. We introduce nominal price rigidity following Rotemberg (1982), by assuming that firms face convex costs of adjusting prices. Specifically, these costs are expressed as follows:

\[ AC_{jt}^Y = \frac{\phi_y}{2} \left( \frac{P_{jt}}{P_{j-1}} - \mu \right)^2 Y_t, \]  

(6)

where \(\phi_y\) measures the degree to which firms dislike to deviate in their price setting behaviour from the steady-state inflation rate \(\mu\). The representative firm chooses a plan for production, with the resulting demand for labour and capital so as to maximize the expected present value of its profits:

\[ \pi_{jt} = E_0 \sum_{t=0}^{\infty} \rho_t P_t \pi_{jt}, \]  

where \(\rho_t\) is a pricing kernel representing the marginal utility value to the representative household of an additional unit of profits accrued in period \(t\): \(\rho_t = \beta' A_t.\) Profits and technology are given by the following expressions:

\[ P_t \pi_{jt} = P_{jt} Y_{jt} - P_t W_t L_{jt} - P_t Z_t K_{jt} - P_t AC_{jt}^Y, \]  

(7)

\[ Y_{jt} = A_t K_{jt}^{1/(1-x)} - \Phi, \]  

(8)

where \(A_t\) describes the economy-wide state of technology at period \(t\) and \(\Phi\) represents a fixed cost, which is calibrated to make profits equal to zero in steady-state equilibrium.

The public sector budget constraint is given by the following equation:

\[ M_t - M_{t-1} + (B_t - R_{t-1} B_{t-1}) = P_t T_t. \]  

(9)

The government derives revenue from issuing money and debt, which it uses to make transfers to the households and to pay interest on outstanding debt. Monetary policy can be described by the following exogenous process of money growth: \(M_t/M_{t-1} = \mu \mu_t\). A shift in monetary policy takes the form

\(^2\) Using this result and the zero profit condition for the aggregator yields the following aggregate price index function:

\[ P_t = \left[ \frac{1}{J} \sum_{j=1}^{J} (P_{jt})^{1-\theta} \right]^{1/(1-\theta)}. \]

\(^3\) Where \(A_t\) represents the marginal utility of consumption.
of an unexpected permanent rise in the money supply: $\mu_t = \gamma \mu_{t-1} \exp\{e_{\mu t}\}$, where $e_{\mu t}$ is a normally distributed i.i.d. zero-mean shock with standard deviation $\sigma_{\mu}. $ Finally, the fiscal policy reaction function has no stochastic component. We specify the following rule in terms of the transfers and real bonds: $P_tT_t = -\tau B_{t-1},$ where $\tau$ is a positive constant. Thus, transfers are determined to maintain the dynamic stability of the model and to make sure that the economy is in a Ricardian regime, in which prices are determined by the monetary policy.

We define a symmetric monopolistic competition equilibrium as:

(a) The set of quantities $(Y_{jt}, C_{it}, I_{it}, L_{it}, K_{it}^x, M_{it}, B_{it})$ that maximizes the constrained present value of the stream of utility of the representative household and the constrained present value of the profits earned by the representative firm.

(b) The set of prices $(P_t, W_t, Z_t, R_t)$ that clears the goods markets, the labour market and the money, bonds and capital markets.

An extensive representation of the symmetric equilibrium is obtained from the first order conditions of both the $i$th household and the $j$th firm. Aggregating over $i$ and $j$ yields a set of equations which define the symmetric equilibrium of the economy (see Appendix A for details).

To solve the model we first write the equilibrium equations in terms of stationary variables, dividing the nominal variables by the money supply ($M$). Second, since an exact expression for the equilibrium cannot be found analytically, we approximate the solution by log-linearizing the equilibrium around the steady state. Then, following Sims (1995), we write the system of transformed equations as $\Gamma_0x_t = \Gamma_1x_{t-1} + \Gamma_2e_{\mu t} + \Gamma_3\zeta_t$, where $x_t$ is the vector of percentage deviations of endogenous variables with respect to their steady state, $e_{\mu t}$ is the monetary policy shock, and the last term $\zeta_t = (x_t - E_{t-1}x_t)$ defines expectational errors. The parameter matrices $\Gamma_i (i = 0, 1, 2, 3)$ are non-linear transformations of the structural parameters.

3. Specification of preferences and calibration

3.1. Preferences

The liquidity effect is related to the change in the marginal utility of consumption and that depends heavily on the cross derivative $U_{CL}$ once we allow for the presence of significant substitution and income effects in the labour supply. We consider three different momentary utility functions that have

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4 This solution method is based on generalized eigenvalue decomposition and it extends the one described by Blanchard and Khan (1980).
been extensively used in the business cycle literature, which differ from each other in relation to the within-period separability between consumption and leisure:

\[ U_{it} = \frac{1}{1 - \sigma_1} \left[ C_{it}^{\sigma_2} + b[M_{it}/P_t]^{\sigma_2} \right]^{(1 - \sigma_1)/\sigma_2} + \kappa \left( 1 - L_{it} \right)^{1+1/\varepsilon} \left( 1 - \varepsilon_1 \right) \]

(10)

\[ U_{it} = \frac{1}{1 - \sigma_1} \left[ C_{it}^{\sigma_2} + b[M_{it}/P_t]^{\sigma_2} \right]^{(1 - \sigma_1)/\sigma_2} \left( 1 - L_{it} \right)^{\psi(1 - \sigma_1)} \]

(11)

\[ U_{it} = \left\{ \left[ C_{it}^{\sigma_2} + b[M_{it}/P_t]^{\sigma_2} \right]^{1/\sigma_2} - \psi_0 L_{it}^{\varepsilon} \right\}^{1-\sigma_1} \]

(12)

The usual restrictions imposed on the parameters ensure that the utility is concave, \( C \) and \( M/P \) are normal goods and the interest elasticity of money demand is strictly negative. We can think of the momentary utility function as depending on a composite good, which is a CES aggregator of consumption and real balances. This allows us to make our exercises across alternative preferences comparable in terms of the specification of the money demand. In particular, the log-linear approximation of the first order conditions yields the following expression for the money demand:

\[ m_t - p_t = c_t - \varepsilon_t r_t, \]

(13)

where \( \varepsilon_t = 1/(1 - \sigma_2)(R - 1) \), and \( R \) is the steady-state gross nominal interest rate and \( m_t, p_t, c_t \) and \( r_t \) represent the log-deviation of money, prices, consumption and the gross nominal interest rate, respectively. The unit elasticity of consumption is consistent with the long run estimate in Lucas (1988).

Eq. (10) represents separable preferences, where \( \sigma_1 > 0 \) characterizes risk aversion and \( \varepsilon \) is the labour supply elasticity. Eq. (11) is a general form of two alternative preference specifications. On the one hand, if we set \( \psi = 1 - a \), we obtain the standard Cobb–Douglas preference specification. On the other hand, if \( a = 1 \) we obtain the preferences used by Chari et al. (2000), from which it is easy to compute the labour supply elasticity. The third class of utility functions (expression (12)) is the one advocated by Greenwood et al. (1988) (GHH, henceforth). These preferences have two properties that may be relevant in understanding the liquidity effect: (i) first, the elasticity of intertemporal substitution of leisure is zero; and (ii) the number of hours worked is a function of the current wage so that there is no income effect on labour supply. The elasticity of labour supply implied by these preferences is \( 1/v - 1 \) (with \( v > 1 \)). Finally, the parameters \( \kappa \) and \( \psi_0 \) will be chosen so that the total hours worked by agents are a given proportion of their time endowment.
Table 1

A. Baseline values for calibration parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount factor</td>
<td>$\beta$</td>
</tr>
<tr>
<td>Preferences</td>
<td></td>
</tr>
<tr>
<td>Risk aversion</td>
<td>$\sigma_1$</td>
</tr>
<tr>
<td>Separable</td>
<td></td>
</tr>
<tr>
<td>Labour supply elasticity</td>
<td>$\varepsilon$</td>
</tr>
<tr>
<td>Non-separable (Greenwood–Hercowitz–Huffman (GHH))</td>
<td></td>
</tr>
<tr>
<td>Labour supply elasticity</td>
<td>$1/(v - 1)$</td>
</tr>
<tr>
<td>Money demand</td>
<td></td>
</tr>
<tr>
<td>Interest rate elasticity ($\varepsilon_r$)</td>
<td>$1/(1 - \sigma_2)(R - 1)$</td>
</tr>
<tr>
<td>Technology and capital accumulation</td>
<td></td>
</tr>
<tr>
<td>Capital income share</td>
<td>$\alpha$</td>
</tr>
<tr>
<td>Depreciation rate</td>
<td>$\delta$</td>
</tr>
<tr>
<td>Capital adjustment cost parameter</td>
<td>$\phi_k$</td>
</tr>
<tr>
<td>Price setting</td>
<td></td>
</tr>
<tr>
<td>Steady-state markup</td>
<td>$\theta/(\theta - 1)$</td>
</tr>
<tr>
<td>Price-adjustment cost parameter</td>
<td>$\phi_y$</td>
</tr>
<tr>
<td>Monetary policy</td>
<td></td>
</tr>
<tr>
<td>Autocorrelation of money-growth shocks</td>
<td>$\rho_n$</td>
</tr>
</tbody>
</table>

B. Steady state

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$</td>
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</tr>
<tr>
<td>$R$</td>
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</tr>
<tr>
<td>$M/B$</td>
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</tr>
<tr>
<td>$L$</td>
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</tr>
<tr>
<td>$C/Y$</td>
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</tr>
<tr>
<td>$K/Y$</td>
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</tr>
<tr>
<td>$I/Y$</td>
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</tr>
<tr>
<td>$\Phi/Y$</td>
<td>0.20</td>
</tr>
<tr>
<td>Price adjust. costs/Y (%)</td>
<td>0.70</td>
</tr>
<tr>
<td>Capital adjust. costs/I (%)</td>
<td>1.74</td>
</tr>
</tbody>
</table>

3.2. Calibration

This section describes the benchmark values used to compute the response of the economy to monetary shocks. These parameters together with the steady state of the economy are reported in Table 1. We set the discount
parameter $\beta$ to $(0.97)^{1/4}$ which implies that the real interest rate is equal to 3 percent per annum. The gross nominal interest rate and the inflation rate are set at 1.08 and 1.05, respectively. Our benchmark value for the risk aversion parameter ($\sigma_1$) is equal to 2. With separable and GHH preferences the elasticity of the labour supply with respect to real wages is set equal to 1. When we use Cobb–Douglas preferences in Eq. (11), we set $\psi = 1 - a$ to ensure that agents work 30 percent of their time endowment. We choose an interest rate elasticity ($\varepsilon_r$) equal to 0.4 in expression (13) consistent with the estimated elasticity of real balances for US obtained by Chari et al. (2000). The labour income share $(1 - \pi)$ is equal to 0.33, and the annual depreciation rate is equal to 10 percent. The capital-adjustment-cost parameter, $\phi_k$, is 10. This value implies that the installation of capital involves a 1.75 percent cost in terms of investment, and a capital–output ratio of 2.5.\footnote{These values are consistent with microeconometric estimates (see, for instance, Whited, 1992).}

We turn now to the consideration of the parameters $\theta, \Phi$ and $\phi_y$. The elasticity of demand is chosen $(\theta = 6)$ such that the markup in a flexible price economy is 20 percent. Assuming zero profits in the steady state, this assumption is equivalent to making the value of $\Phi/Y = (1/\theta)A(K^{\pi}L^{1-\pi})$ also equal to 20 percent. Although these values are conventional in the literature, our results do not change when a lower markup is used (say, 5 percent) as suggested by the evidence in Basu and Fernald (1997). The price-adjustment-cost parameter ($\phi_y$) cannot be calibrated using steady-state information. To choose a value we follow recent estimates of the new-Keynesian Phillips trade-off. In particular, the slope of the Phillips curve is given by $(\theta - 1)/\phi_y$. Thus, prices adjustments are more rapid when $\theta$ increases (more competition in the goods market) and when $\phi_y$ decreases (lower adjustment costs). Setting $\phi_y = 85$ makes that ratio equals to 6 percent. This value is in line with recent estimates by Sbordone (1998) and is approximately what comes out under the calibration of Calvo staggered prices assuming that on average prices remain fixed during four quarters (e.g. Erceg et al., 2000). Finally, there is a debate about the monetary policy that best represents the endogenous policy rule followed by the central bank. Christiano et al. (1998) show that the US monetary policy rule can be reasonably well approximated with an AR(1) specification, with $\rho_\mu = 0.5$ when money growth is measured in terms of M2.

4. The effects of permanent unanticipated money supply shocks

In this section, we assess the role of preferences, capital accumulation and other features of our model in shaping the response of interest rates to monetary shocks: the liquidity effect. We proceed step-by-step, first working
with a version of the model with separable preferences, then introducing non-separable preferences and finally adding capital into the model. We show that for a given exogenous money-growth rule the existence of a liquidity effect depends critically on the interaction of these elements with the rest of the model. With the complete model we analyse the properties of the sets of parameters that generate the liquidity effect, discussing the importance of the real and nominal rigidities and of the persistence of the stochastic monetary process.

4.1. Economy without capital: Separable preferences

Let us consider a simple version of the model presented in Section 2, where preferences are separable, as expressed by (10), and without capital, so that the production function is defined as \( Y_{jt} = A_t L_{jt}^{1-\sigma} - \Phi \). Fig. 1A shows that positive and persistent monetary shocks generate a positive and persistent response of consumption, real balances and output. Since prices are sticky, a positive nominal shock increases demand and marginal costs but lowers the markup. Labour demand, output and consumption increase. Following the rise in real wages, labour supply also increases, pushing the economy closer to the equilibrium that would prevail with flexible prices. Eventually, the price level adjusts to its new level, reverting the initial reduction in the markup and returning the economy to its steady state.

These real effects occur alongside a positive impact on the nominal interest rate. This impact effect is characterized by two equilibrium equations representing the intertemporal allocation of consumption and the demand for money balances:

\[
\frac{U_{ct}}{P_t} = \beta R_t E_t \left( \frac{U_{ct+1}}{P_{t+1}} \right),
\]

\[
\frac{U_{mt}}{U_{ct}} = (1 - 1/R_t),
\]

where Eq. (14) represents the intertemporal Euler equation for consumption, and Eq. (15) determines the optimal allocation between consumption and real balances within each period (i.e. it represents a money demand equation). Christiano et al. (1997) combine Eq. (14) with a cash-in-advance constraint yielding an expression of the type \( C_t U_{ct} = \beta R_t E_t ((C_{t+1} U_{ct+1})/(M_{t+1}/M_{t+2})) \). These authors show that the expectational term becomes a constant (say, \( S \)) in the event that prices are set one period in advance and money growth follows an i.i.d. process. Under these assumptions, after a positive money-growth shock, the only way to generate a liquidity effect is through a reduction in \( C_t U_{ct} \). With separability between consumption and leisure, \( R_t = C_t^{1-\sigma_t}/S\beta; \)

\textsuperscript{6} These are derived from expression (A.1), (A.3) and (A.4) in Appendix A.
Fig. 1. Panel A: Impulse response to a persistent money growth shock; separable preferences without capital. Parameter values: $\sigma_1 = 2$, $\epsilon = 1$, $\phi_y = 85$, $\rho_\mu = 0.5$. Panel B: Robustness to changes in the risk aversion. Parameter values: $\epsilon = 1$, $\phi_y = 85$, $\rho_\mu = 0.5$. 
hence, after an increase in the money supply that leads to a rise in $C_t$, the nominal interest rate will fall if and only if the risk aversion parameter is $> 1$ ($\sigma_1 > 1$).

Unfortunately, this result cannot be so easily obtained once we relax some of Christiano, Eichenbaum and Evans’s assumptions regarding price setting and the money-growth process. Nevertheless, we can still log-linearize the above expressions (14) and (15) around the steady state to obtain

$$r_t - E_t \Delta p_{t+1} = \sigma_1 (E_t c_{t+1} - c_t),$$

(14a)

where lower case letters represent log-deviations from their steady-state value. Using the previous expression and Eq. (13) allows us to solve for the nominal interest rate under the simplifying assumption that money demand only depends on consumption ($\varepsilon_w = 0$, i.e. $\sigma_2 = -\infty$):\(^7\)

$$r_t = \sigma_1 \left[ E_t \Delta m_{t+1} + \left( \frac{1}{\sigma_1} - 1 \right) E_t \Delta p_{t+1} \right].$$

(16)

For a given degree of money and price persistence the monetary shock generates an impact increase in money and prices that is expected to continue in the future. Under these fairly general circumstances, and for a given value of $\rho_\mu$, a high risk aversion (i.e. a low intertemporal elasticity of substitution $(1/\sigma_1)$) is needed to obtain the liquidity effect.

The intuition behind this result can also be cast in terms of the impact effect on the right-hand side of expression (13a), i.e. the real interest rate. After a positive monetary shock, consumption rises at time $t$ and declines from $(t + 1)$ onwards (see Fig. 1A). Thus, the impact effect on the real interest rate is negative and is given by $\sigma_1 = 2$ times the expected decrease in consumption. Since consumption is the only argument of marginal utility, it is the expected change in consumption that must be strong enough to compensate for the increase in inflation expectations and generate the liquidity effect. Thus, increasing the risk aversion tends to produce a liquidity effect by increasing the change in the real interest rate required to accommodate a given intertemporal consumption profile.\(^8\)

To put some numbers to this result, we simulate in Fig. 1B the impact effect on the nominal interest rate and real balances of a money-growth shock for

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\(^7\) When $\varepsilon_t$ is different from zero we solve forward for the nominal interest rate and obtain:

$$r_t = f \left\{ \sum_{j=0}^{\infty} \delta^j \left[ E_t \Delta m_{t+j+1} + \left( \frac{1}{\sigma_1} - 1 \right) E_t \Delta p_{t+j+1} \right] \right\} \text{ where } \delta = \frac{\varepsilon_t}{1/\sigma_1 + \varepsilon_t};$$

$$f = \left( \frac{1}{\sigma_1} + \varepsilon_t \right)^{-1}.$$

\(^8\) In a cash-in-advance economy without capital and one-period price stickiness, Jeanne (1994) also gets this result.
different values of the risk aversion parameter. This figure confirms the above
surmise. Thus, in sticky price models if preferences are separable, the higher
the risk aversion the more likely there will be a liquidity effect. Notice that
the existence of price and money-growth persistence means that a high degree
of risk aversion is needed to generate the liquidity effect. If, instead, we were
assuming a lower interest rate elasticity of real balances, then a much higher
value of the risk aversion parameter would be needed to generate that effect.

4.2. Economy without capital: Non-separable preferences

The above results do not hold under non-separable preferences. In partic-
ular, expression (16) no longer represents the interest rate response, since
the intertemporal allocation of consumption depends on other features of the
model. Fig. 2A shows the response of the main variables in the model to a
persistent monetary shock, both under Cobb–Douglas preferences and GHH
preferences. At first glance, the impulse response functions look very similar
in this economy as compared with those in the model with separable prefer-
ences. A positive monetary shock increases consumption and real balances,
while the nominal interest rate still increases on impact. Nevertheless, a closer
look at the results shows an interesting departure from the general proposition
enunciated above. As the sensitivity analysis in Fig. 2B makes clear higher
risk aversion does not longer guarantee the existence of the liquidity effect.

To give some intuition for this latter result, we consider the case of non-
separable preferences given by expression (11), when \( a = 1 \). The log-linear
equation for the intertemporal consumption allocation (14) now takes the
form

\[
rt - E_t \Delta pt+1 = \left( \frac{\bar{L}}{1 - \bar{L}} \right) (1 - \sigma_1)(E_t \ell_{t+1} - \ell_t) + \sigma_1(E_t c_{t+1} - c_t).
\]

(14b)

The real interest rate moves so as to equalize the expected change in the
marginal utility of consumption between \( t \) and \( t + 1 \), but now the ratio of
marginal utilities not only depends on the expected change in consumption
but also on the expected response of the labour supply. Hence increasing the
risk aversion has now two counteracting effects upon the change in the real
interest rate required to accommodate a joint intertemporal profile of both \( c_t \)
and \( \ell_t \), and does not help to obtain the liquidity effect.

Moreover, in this case the liquidity effect is never obtained for positive
values of \( \sigma_1 \). This result relies heavily on the fact that, in a model without
capital, consumption is proportional to labour (i.e. \( c_t \approx a \ell_t \)). Thus, for the
benchmark values of the parameters, the expected increase in consumption
and leisure roughly offset each other in expression (14b) leaving the real
Fig. 2. Panel A: Impulse response to a persistent money growth shock; Cobb–Douglas (CD) and GHH preferences without capital. Parameter values: $\sigma_1 = 2$, $\psi = 1 - a = 0.60$, $v = 2$, $\phi_y = 85$, $\rho_t = 0.5$. Panel B: Robustness to changes in the risk aversion; comparing Cobb–Douglas (CD) with GHH preferences. Parameter values: $\psi = 1 - a = 0.60$, $v = 2$, $\phi_y = 85$, $\rho_t = 0.5$. 
interest rate almost unchanged, as we move towards higher values of the risk aversion parameter.

As noted in Section 3, the previous preferences (expression (11) with \( a = 1 \)) impose strong income and substitution effects on leisure after positive monetary shocks. Thus, we now analyse how allowing for a zero intertemporal substitution effect on leisure and a zero income effect on labour supply affects our previous result. GHH preferences entail the following (log-linear) expression for the intertemporal consumption allocation:

\[
(1 - \eta/v)(r_t - E_t \Delta p_{t+1}) = \sigma_1[(E_t c_{t+1} - c_t) - \eta(E_t \ell_{t+1} - \ell_t)],
\]

where \( \eta = (1 - \alpha)/(c/y) \). With these preferences households smooth \( (c_t - \eta \ell_t) \) over time instead of \( c_t \). The absence of an income effect on the labour supply makes the movements in consumption and leisure roughly proportional and this is independent of the degree of intertemporal substitution. As shown in Fig. 2B, increases in risk aversion affect the nominal interest rate more significantly than in the previous preferences, but it is never sufficient to obtain a negative impact effect.

Since this is a most unfortunate feature of the model we may ask at this point what it takes to obtain a proper liquidity effect in a sticky price model with non-separable preferences. What we need is a mechanism that drives the response of labour supply and consumption significantly apart so that the real interest rate can fall substantially on impact.

### 4.3. Economy with capital: Non-separable preferences

Capital accumulation is a key feature of the model that may have a major effect on the way some endogenous variables respond to a monetary shock. In particular, current consumption can be made less responsive to the money shock for a given output response, since households devote part of their income to buy capital goods (in addition to bonds and real balances). We will show that in order to generate the liquidity effect the mere presence of capital is not enough, what is needed is a strong incentive to accumulate it. This can be obtained in the previous model if the risk aversion and the adjustment costs of capital \( (\phi_k) \) are low.

Fig. 3A shows the impulse-response of an economy with capital accumulation. In the case of high risk aversion \( (\sigma_1 = 2) \), the incentive to save and accumulate capital is low, and therefore the allocation of both real balances and consumption are very similar to those obtained in the model without capital. Thus, the equilibrium generates the same path for interest rates: an increase on impact followed by a smooth decrease over time. Nevertheless, the money-shock simulation when risk aversion is low \( (\sigma_1 = 0.65) \) generates a large substitution effect that is reflected in huge output and labour responses on impact but only a small change in consumption. For \( \sigma_1 = 0.65 \), the interest
Fig. 3. Panel A: Impulse response to a persistent money growth shock; non-separable preferences (Cobb–Douglas) with capital. Parameter values: $\sigma = 2$, $\psi = 1 - a = 0.67$, $\phi = 85$, $\phi_k = 10$, $\rho_\mu = 0.5$. Panel B: Robustness to changes in the risk aversion; comparing Cobb–Douglas (CD) with GHH preferences. Parameter values: $\psi = 1 - a = 0.67$, $\phi = 85$, $\phi_k = 10$, $\rho_\mu = 0.5$. 
rate response generates a liquidity effect. We can explain the impact response of interest rates in terms of expression (14b): if the risk aversion parameter is \( \sigma_1 < 1 \), then there is a large expected fall in labour after the first period that brings current interest rates down.\(^9\)

Fig. 3A also highlights the role of capital in this model. When \( 1/\sigma_1 \) is large, consumers have the incentive to postpone consumption which, unlike the model without capital, does not necessarily involve postponing production (and so employment) too. On the contrary, as the real rate falls, the initial jump in the expected shadow price of capital leads to a sharp increase in the demand for investment and, hence, in output and labour demand.\(^10\) As can be seen in expression (14b), the sharp expected reduction in labour \( (E_t \Delta L_{t+1}) \) now more than offsets the expected consumption increase \( (E_t \Delta c_{t+1}) \) to produce a strong fall in the real interest rate. Why do households postpone consumption and work harder today despite the fact that the real interest rate has fallen? The reason is that there is a new asset, capital, which makes it very profitable to devote any additional amount of resources to its accumulation. As \( \sigma_1 \) gets bigger, all these effects become smaller and the economy resembles very much the one without capital in which the movements in consumption do not dominate the movements in labour, so leaving no room for a fall in the nominal interest rate.

To sum up, when preferences are Cobb–Douglas and there is capital accumulation the liquidity effect is more likely the smaller the risk aversion parameter, just the opposite of what happens in an economy with separable preferences. Although Cobb–Douglas preferences are the only ones of three considered that satisfy balance growth, the range of values for which such an effect exists is very small and is always for risk aversion values lower than one (see Fig. 3B). That generates implausibly large movements in output and labour, whereas consumption remains fairly stable.

To get a more complete picture of the features shaping the initial response of the nominal interest rate in this model, we check the robustness of our results with respect to the remaining parameters of the model. Fig. 4 depicts the results of sensitivity analyses with regard to nominal rigidities, real rigidities and the persistence of the monetary shock for different values of the risk aversion parameter under Cobb–Douglas preferences.\(^11\) In panel A, monetary shocks are represented by shocks to the rate of money growth with different degrees of persistence \( (\rho_m) \), and the contour levels represent the impact effect on the nominal interest rate for different combinations of \( \phi_k \) and \( \rho_m \).

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\(^9\) A similar reasoning applies to a high value of the capital adjustment costs \( (\phi_k) \).

\(^10\) Notice that the additional output demand in period \( t \) must be entirely met by a rise in employment since \( K_t \) is predetermined.

\(^11\) We do not report the response of real money balances that is positive for all parameter combinations considered.
Fig. 4. Sensitivity analysis: impact effect on nominal interest rates (Cobb–Douglas preferences in a model with capital). Panel A: Shocks to money growth. Panel B: Shocks to the level of money.
and $\phi_y$ and $\rho$, respectively. Within the high risk aversion region (graphs on the left), the impact effect of the interest rate becomes smaller as the cost of adjusting capital rises ($\phi_k$), for a given persistence of the money shock ($\rho$). Also, for a given $\phi_k$, the response of the nominal rate is smaller for low values of $\rho$. The contours in the space $\phi_y$, $\rho$ show that the impact response of the nominal rate is, as expected, smaller as we move towards the origin. Nevertheless, within the region of high risk aversion, the chances of obtaining the liquidity effect are remote.

The liquidity effect is more likely in the region of low risk aversion (graphs on the right), but still whether or not the interest rate falls on impact depends on other parameters of the model. Interestingly, the contours change slope with respect to the high risk aversion graphs. When $\sigma_1 = 0.65$, the probability of obtaining a negative response by the nominal rate increases as we move from high to low values of $\rho$, but the liquidity effect becomes more likely for low, rather than for high, values of $\phi$. The cost of adjusting capital is borne by households and the lower this cost the higher the incentive to accumulate capital, leading to the stronger output and employment responses needed to guarantee the liquidity effect. Similarly, the interest rate falls more the higher the value of $\phi_y$, i.e. the liquidity effect is more likely the higher the degree of price inertia. The impact response of the nominal interest rate is always higher the higher the value of $\rho$. Dow (1995) analyses the response of nominal interest rates in a model without inflation expectations effect ($\rho = 0$). In such a world, Fig. 4 shows that, for low capital adjustment cost, there is always a liquidity effect.$^{12}$

When the monetary shock affects the level of the money supply (with persistence $\rho_m$) instead of its rate of growth (panel B), the liquidity effect is obtained more often. Although the slope and the ordering of the contours remain close to those in panel A, the impact response of the nominal rate is negative for values of $\rho_m$ that are lower than $\sim 0.5$ even when $\sigma_1$ takes a large value. Overall, these robustness exercises confirm that, under non-separability and capital accumulation, the risk aversion parameter is, along with the degree of persistence of the monetary shock, crucial in shaping interest rate movements.

5. Conclusions

Although neither the output nor the price effects of a monetary expansion need to be preceded by a fall in the nominal interest rate, the liquidity effect is viewed by many economists as one of the well established empirical facts

\footnote{In Dow’s model the adjustment cost of capital is borne by the firms. There, the higher the adjustment costs the more likely is to produce a decrease on the interest rates.}
in monetary economics. General equilibrium models aiming to represent this mechanism face the challenge of reproducing the liquidity effect along with other business cycle features of market economies. In this paper, we have discussed under what conditions a general equilibrium model with costs of adjusting prices and capital is capable of generating a downward movement in the nominal interest rate, following a positive monetary innovation.

In a world without capital and with separable preferences, the logic of the intertemporal allocation of wealth leads to the following result: it takes a high risk aversion parameter for the nominal interest rate to fall on impact. This is so because the marginal utility of consumption is only driven by the dynamics of consumption. When preferences are not separable the previous result does not apply. In this case, the impact response of nominal interest rate is not always sensitive to changes in the intertemporal elasticity of substitution, since the movements of the marginal utility of consumption and labour almost compensate each other.

In an economy with non-separable Cobb–Douglas preferences and capital accumulation, the liquidity effect is not obtained unless the risk aversion parameter is very low, and provided that the capital adjustment costs, borne by households, are not high. In such a case, the intertemporal substitution effect leads households to postpone consumption and invest in capital, increasing current output and labour by an amount large enough to reduce the real interest rate by more than the expected increase in prices.

The reason why it takes a low $\sigma_1$ and low $\phi_k$ to produce the liquidity effect in our model can be best understood by exploiting the similarity of the monetary transmission mechanism with such parameters to that implied in the simplest limited participation model. In that model, restrictions on the adjustment of consumers’ portfolios make it possible that the additional liquidity of the system will not increase current consumption. The additional savings are thus devoted to increasing the supply of productive factors, which firms are willing to hire and use in production if the interest rate falls. In sticky price models, in which the marginal utility of consumption depends upon consumption and leisure, we can obtain a similar result by imposing a strong preference towards future consumption (high intertemporal elasticity of substitution or very low $\sigma_1$) and low capital-adjustment costs. This produces high savings and a high incentive to accumulate and supply capital, which induces a large enough fall in the real interest rate. Nevertheless, in this latter model this comes at the cost of generating implausibly large impact responses by output, employment and investment.

Finally, the impact response of the nominal interest rate also depends on other parameters of the model, such as the degree of nominal (price) and real (capital adjustment costs) inertia and the process assumed for the monetary shock. Nevertheless, these parameters are of secondary importance, compared with to preferences and the money stock process, as regards the ability of
the model to generate the liquidity effect. Thus, we conclude that accounting for the observed liquidity effect in sticky price models is not straightforward and it may require additional restrictions on the preferences and/or alternative specifications of the monetary policy.

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Appendix A. An extensive representation of the symmetric monopolistic competition equilibrium

The household’s choice is given by

\[
\frac{\partial U}{\partial C_t} = A_t P_t, \tag{A.1}
\]

\[
\frac{\partial U}{\partial L_t} = -A_t P_t W_t, \tag{A.2}
\]

\[A_t = \beta E_t R_t A_{t+1}, \tag{A.3}\]

\[
\frac{\partial U}{\partial M_t} = A_t - \beta E_t A_{t+1}. \tag{A.4}
\]

Optimal capital accumulation is derived from the first order conditions of the households’ optimization problem with respect to investment and capital:

\[
\beta(1 - \delta)E_t q_{t+1} - q_t + A_t \left[ P_t Z_t + P_t \phi_k \left[ \frac{I_t}{K_t} \right]^3 \right] = 0, \tag{A.5}
\]

\[A_t P_t \left[ 1 + \frac{3\phi_k}{2} \left[ \frac{I_t}{K_t} \right]^2 \right] = \beta E_t q_{t+1}. \tag{A.6}\]
The first order conditions of the firm with respect to employment and capital may be written as

\[ W_t = (1 - \alpha) \left[ \frac{Y_t + \Phi}{L_t} \right] \left[ 1 - (1/e_{yt}) \right], \quad (A.7) \]

\[ Z_t = \alpha \left[ \frac{Y_t + \Phi}{K_t} \right] \left[ 1 - (1/e_{yt}) \right], \quad (A.8) \]

where \( e_{yt} \) satisfies the following expression:

\[ e_{yt} = \theta \left\{ 1 - \phi_y \left( \frac{P_t}{P_{t-1}} \right) \left[ \frac{P_t}{P_{t-1}} - \mu \right] \right. \\
\quad + \left. \phi_y E_t \left[ \frac{P_{t+1}}{\rho_t} \left( \frac{P_{t+1}}{P_t} \right)^2 \frac{Y_{t+1}}{Y_t} \left( \frac{P_{t+1}}{P_t} \right) \right] \right\}^{-1}. \quad (A.9) \]

Every agent has access to a complete and competitive market for contingent claims. This is equivalent to say that firms maximize their market value and that the unique real discount factor satisfies

\[ \left\{ \frac{P_{t+1}}{\rho_t} \right\} = \beta \left\{ \frac{A_{t+1}}{A_t} \right\}. \quad (A.10) \]

Finally, the government budget constraint, capital accumulation and the production function are

\[ M_t - M_{t-1} + (B_t - R_{t-1}B_{t-1}) = P_t T_t, \quad (A.11) \]

\[ K_t = I_{t-1} + (1 - \delta)K_{t-1}, \quad (A.12) \]

\[ Y_t = A_t L_t^{1-\gamma}K_t^\gamma - \Phi. \quad (A.13) \]

Using these, the household budget constraint and the definition of profits,

\[ \pi_t = Y_t - W_t L_t - Z_t K_t - AC_t^Y, \]

we derive the economy wide constraint:

\[ C_t + I_t \left( 1 + \frac{\phi_k}{2} \left( \frac{I_t}{K_t} \right)^2 \right) = Y_t - AC_t^Y. \quad (A.14) \]

Price adjustment costs satisfy the following quadratic expression:

\[ AC_t^Y = \frac{\phi_y}{2} \left( \frac{P_t}{P_{t-1}} - \mu \right)^2 Y_t. \quad (A.15) \]

We specify the following fiscal policy in terms of the transfers:

\[ P_t T_t = - \tau B_{t-1}, \quad (A.16) \]

where \( \tau \) is a positive constant. Thus, transfers are determined to maintain the dynamic stability of the model. The equation for the monetary policy
completes the system of 17 dynamic equations with 17 endogenous variables: prices ($P_t, W_t, Z_t, A_t, \rho_t, q_t, R_t$), quantities ($Y_t, C_t, I_t, L_t, T, M_t, B_t, K_t$), the markup $\left( (1 - (1/\epsilon_Y))^{-1} \right)$ and $AC_Y^t$.

References


