Problemas propuestos de algebra Booleana y puer. lógicas (4)

(4.1) Funciones + y · asociativas. Comprobarlo con la T.V.

$S = a + b + c \Rightarrow S = (a+b)+c$

a	b	c	S
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

a	b	$s' = a+b$
0	0	0
0	1	1
1	0	1
1	1	1

s'	c	$S = (a+b)+c$
0	0	0
0	1	1
1	0	1
1	1	1

Si "a" y "b" y "c" son $\emptyset \Rightarrow$

salida $= \emptyset$

(4.2) Aplicar teorema de Shannon a la expresión $\overline{a + b \cdot \bar{c}}$ y comprobar la igualdad con la T.V.

$S = \overline{a + b\bar{c}} \xrightarrow{\text{Morgan}} S = \bar{a} \cdot \overline{b\bar{c}} \xrightarrow{\text{Morgan}} \bar{a} \cdot (\bar{b} + c) \longrightarrow$

$\xrightarrow{\text{distrib}} \bar{a}\bar{b} + \bar{a}c$

Shannon $\rightarrow S = \bar{a}\bar{b} + \bar{a}c = \bar{a}\bar{b}(c + \bar{c}) + \bar{a}c(b + \bar{b}) =$

$$S = \underbrace{\bar{a}\bar{b}\bar{c}}_{m_0} + \underbrace{\bar{a}\bar{b}c}_{m_1} + \underbrace{\bar{a}bc}_{m_3} + \underbrace{\bar{a}\bar{b}c}_{m_1}$$

a	b	c	S (canónica)	$S = \overline{a + b\bar{c}}$
0	0	0	1	1
0	0	1	1	1
0	1	0	0	0
0	1	1	1	1
1	0	0	0	0
1	0	1	0	0
1	1	0	0	0
1	1	1	0	0

Si $a = \emptyset$ y $b = 1$ y $c = \emptyset$ entonces $a + b\bar{c} = 1 \Rightarrow S = 0$. Resto $S = 1$

En S si $a = 1 \Rightarrow S = \emptyset$

AF41

(4.3) Teorema de la expansión

$$S = abc + a\bar{b}c + \bar{a}bc = a(\bar{b}c + bc) + \bar{a}(bc)$$

(4.4) Logigrama

$$F = \Sigma(m1, m3, m6, m7) + x(2,5)$$

	c	b	a	F
0	0	0	0	0
1	0	0	1	1
2	0	1	0	x
3	0	1	1	1
4	1	0	0	0
5	1	0	1	x
6	1	1	0	1
7	1	1	1	1

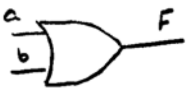
	$\bar{c}$		c	
	$\bar{b}$	b	b	$\bar{b}$
$\bar{a}$	m_0	m_2	m_6	m_4
a	m_1	m_3	m_7	m_5

	$\bar{c}$		c	
	$\bar{b}$	b	b	$\bar{b}$
$\bar{a}$		x	1	
a	1	1	1	x

$1 \rightarrow a$

$2 \rightarrow b$

$$F = a + b$$



(4.5) TV y logigrama de $f(c,b,a) = \Sigma(0,6,7) + x(1,2,5)$

	c	b	a	F
0	0	0	0	1
1	0	0	1	x
2	0	1	0	x
3	0	1	1	0
4	1	0	0	0
5	1	0	1	x
6	1	1	0	1
7	1	1	1	1

	$\bar{c}$		c	
	$\bar{b}$	b	b	$\bar{b}$
$\bar{a}$	1	x	1	
a	x		1	x

$1 \rightarrow bc$

$2 \rightarrow \bar{b}\bar{c}$

$$F = bc + \bar{b}\bar{c} \Rightarrow F = \overline{b \oplus c}$$

4.6 TV y circuito lógico de $f(c,b,a) = \Pi(3,4) \cdot X(2,5,6)$

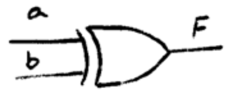
m	M	c	b	a	F
0	7	0	0	0	1
1	6	0	0	1	x
2	5	0	1	0	x
3	4	0	1	1	0
4	3	1	0	0	0
5	2	1	0	1	x
6	1	1	1	0	1
7	0	1	1	1	1

	$\bar{c}$		c	
	$\bar{b}$	b	b	$\bar{b}$
$\bar{a}$	1	x	1	0
a	x	0	1	x

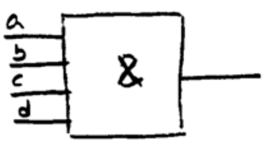
$1 \to bc$

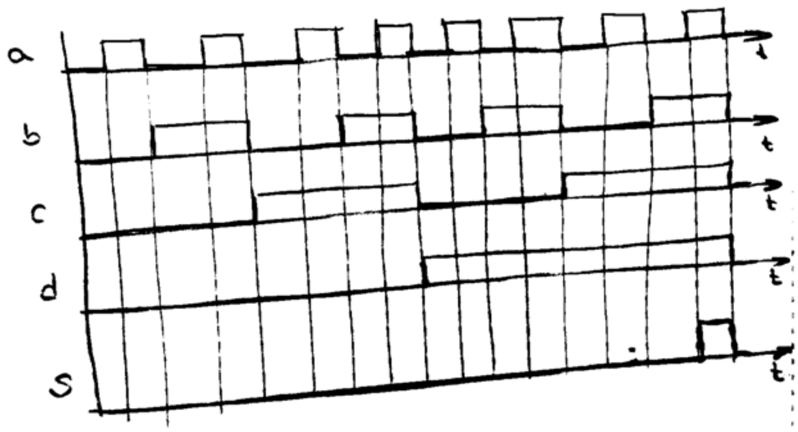
$2 \to \bar{b}\bar{c}$

$$F = bc + \bar{b}\bar{c} \Rightarrow F = \overline{b \oplus c}$$

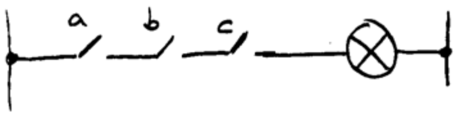


4.7 Cronograma de una AND de 4 entradas





4.8 Circuito eléctrico AND con 3 interruptores

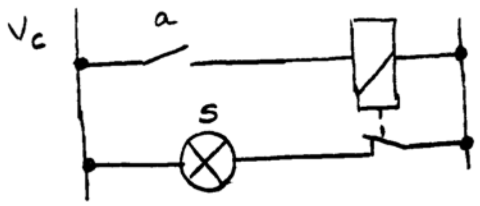


4.9 Logigrama para una alarma que activa una sirena (s) cuando esté activada la señal a y se produzca la rotura de un cristal (b)

a	b	s
0	0	0
0	1	0
1	0	0
1	1	1

$s = a \cdot b$

4.11 Circuito eléctrico con relés para la función NOT



4.14 Diseñar un generador de paridad par para 4 bits

a	b	c	d	P
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	0
0	1	0	0	1
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	1
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	1	0

		$\bar{d}$		d	
		$\bar{b}$	b	b	$\bar{b}$
$\bar{c}$	$\bar{a}$		1		1
	a	1		1	
c	a				1
	$\bar{a}$	1		1	

3 bits $\Rightarrow f(b,c,d) = \bar{b}\bar{c}d + \bar{b}c\bar{d} + b\bar{c}\bar{d} + bcd$

$f(b,c,d) = \bar{b}(\bar{c}d + c\bar{d}) + b(\bar{c}\bar{d} + cd) =$

$f(b,c,d) = \bar{b}(c \oplus d) + b\,\overline{(c \oplus d)} =$

$f(b,c,d) = b \oplus (c \oplus d)$

4 bits Para $a=0 \Rightarrow f(b,c,d)$

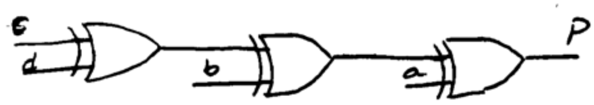
" $a=1 \Rightarrow \overline{f(b,c,d)}$

Si $f(b,c,d) = m \Rightarrow$

$f(a,b,c,d) = \bar{a}m + a\bar{m} = a \oplus m \Rightarrow P = a \oplus (b \oplus (c \oplus d))$

AF14

$P = a \oplus (b \oplus (c \oplus d))$



(4.15) Generador de paridad de 8 bit

De acuerdo al razonamiento de 4.14 $\Rightarrow$

$P = a \oplus (b \oplus (c \oplus (d \oplus (e \oplus (f \oplus (g \oplus h))))))$

Problemas resueltos Gestión (Álg. Boole)

④ ① Minterm L1 según TV

$L1 = m_0 + m_1 + m_2 + m_3 + m_4 + m_5 + m_6 \Rightarrow b$

② Minterm L2 según TV

$L2 = m_3 + m_4 + m_5 + m_6 + m_7 + m_8 + m_9 + m_{10} + m_{11} + m_{12} \Rightarrow d$

③ Minterm L3 según TV

$L3 = m_9 + m_{10} + m_{11} + m_{12} + m_{13} + m_{14} + m_{15} \Rightarrow a$

		$\bar{a}\,\bar{c}$	$\bar{a}\,c$	$a\,c$	$a\,\bar{c}$
$\bar{b}$	$\bar{d}$	m_0	m_2	m_{10}	m_8
	d	m_1	m_3	m_{11}	m_9
b	d	m_5	m_7	m_{15}	m_{13}
	$\bar{d}$	m_4	m_6	m_{14}	m_{12}

L1

		$\bar{a}\,\bar{c}$	$\bar{a}\,c$	$a\,c$	$a\,\bar{c}$
$\bar{b}$	$\bar{d}$	1	1	0	0
	d	1	1	0	0
b	d	1	0	0	0
	$\bar{d}$	1	1	0	0

L2

		$\bar{a}\,\bar{c}$	$\bar{a}\,c$	$a\,c$	$a\,\bar{c}$
$\bar{b}$	$\bar{d}$	0	0	1	1
	d	0	1	1	1
b	d	1	1	0	0
	$\bar{d}$	1	1	0	1

$L1 = \bar{a}\,\bar{c} + \bar{a}\,\bar{b} + \bar{a}\,\bar{d}$

④ $L1 = \bar{a}\,(\bar{c} + \bar{b} + \bar{d}) \Rightarrow \underline{\underline{b}}$

⑤ $L2 = \bar{a}\,b + a\,\bar{b} + \bar{a}\,c\,d + b\,\bar{c}\,\bar{d}$

$\Downarrow$

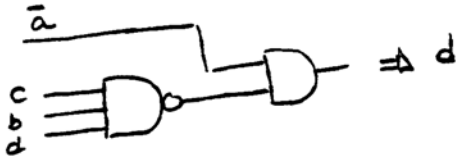
$\underline{\underline{c}}$

L3

		$\bar{a}\,\bar{c}$	$\bar{a}\,c$	$a\,c$	$a\,\bar{c}$
$\bar{b}$	$\bar{d}$	0	0	1	0
	d	0	0	1	1
b	d	0	0	1	1
	$\bar{d}$	0	0	1	1

⑦ $L1 = \bar{a}\,\overline{\overline{(\bar{c} + \bar{b} + \bar{d})}} =$

$L1 = \bar{a}\,\overline{(c \bullet b \bullet d)}$



⑥ $L3 = a\,c + a\,d + a\,b = a\,(b + c + d)$

$\Downarrow$

c

Funciones lógicas

① $F = \overline{\overline{(a+b)}c + \bar{a} + \bar{b}d + \bar{c}\bar{d}} = \overline{\overline{a+b}\,c} \cdot \bar{\bar{a}} \cdot \overline{\bar{b}d} \cdot \overline{\bar{c}\bar{d}} =$

$F = ((a+b)+\bar{c}) \cdot a \cdot (b+\bar{d}) \cdot (c+d) \Rightarrow a$

② La función de NOR exclusiva si O exclusiva $= a\bar{b} + \bar{a}b$

Nor $\text{exclu} = \overline{a\bar{b} + \bar{a}b} = \overline{a\bar{b}} \cdot \overline{\bar{a}b} = (\bar{a}+b)(a+\bar{b}) =$

$= \underbrace{\bar{a}a}_{0} + \bar{a}\bar{b} + ab + \underbrace{b\bar{b}}_{0} = \bar{a}\bar{b} + ab \Rightarrow \underline{c}$

③ $f(a,b,c,d,e) = (ab+\bar{c})(\overline{d+e} + b\bar{a})$ ¿$\overline{f(a,b,c,d,e)}$?

$\overline{f(a,b,c,d,e)} = \overline{(ab+\bar{c})(\overline{d+e} + b\bar{a})} = \overline{ab+\bar{c}} + \overline{\overline{d+e} + b\bar{a}} =$

$= (\overline{ab} \cdot c) + [(d+e) \cdot \overline{b\bar{a}}] = [(\bar{a}+\bar{b})c] + [(d+e) \cdot (\bar{b}+a)]$

$\Downarrow$

$\underline{b}$

④ simplificar $f(a,b,c) = (a+b)(a+c)(b+c) =$

$f(a,b,c) = (\underbrace{\underbrace{\underbrace{aa}_{a} + ac + ab}_{a(1+c+b)}}_{a} + bc)(b+c) = (a+bc)(b+c) =$

$f(a,b,c) = ab + ac + \underbrace{\underbrace{bbc}_{bc} + \underbrace{bcc}_{bc}}_{bc} = ab + ac + bc$

al revés

a) $\underline{a} + c + \underline{ac} + b + c = a + b + c \Rightarrow$ No

b) $(b+c)(a+bc) = ba + bbc + ac + bcc = ab + ac + bc \Rightarrow \underline{\text{SI}}$

⑤ $f(a,b,c,d,e) = (\bar{a} \cdot \bar{b} + d)(\bar{b} + e) + c\bar{d}$ para $a = c = e = \emptyset$ y $b = d = 1$ es

$f(a,b,c,d,e) = (1 \cdot 0 + 1)(0 + 0) + 0 \cdot 0 = \emptyset \Rightarrow \underline{a}$

⑥ $f(a,b,C) = ab + \bar{b} = (a + \bar{b})(b + \bar{b}) = a + \bar{b} \Rightarrow \underline{b}$

⑦ $f(a,b,c) = \bar{a} + \bar{c} =$

b) $f(a,b,c) = \bar{a}\bar{b}\bar{c} + \bar{a}\bar{b}c + ab\bar{c} + \bar{a}b\bar{c} + a\bar{b}\bar{c} + \bar{a} + \bar{c} + abc$

$= \bar{a}(1 + \bar{b}\bar{c} + \bar{b}c + b\bar{c}) + \bar{c}(1 + ab + a\bar{b}) + abc =$

$= \bar{a} + \bar{c} + abc \Rightarrow$ No

d) $f(a,b,c) = \bar{a} + a\bar{b}\bar{c} + ab\bar{c} = \bar{a} + a\bar{c}(b + \bar{b}) = \bar{a} + a\bar{c} =$

$= (\bar{a} + a)(\bar{a} + \bar{c}) = \bar{a} + \bar{c} \Rightarrow$ **SI**

Simplificación de funciones

① $f(a,b) = \overline{a + a\bar{b}} = \overline{a(1 + \bar{b})} = \bar{a} = \bar{a}b + \bar{a}\bar{b} = m_1 + m_0$

$\Downarrow$

$\underline{b}$

② $f(a,b) = \overline{a + a\bar{b}} = m_1 + m_0 =$

$\overline{f(a,b)} = m_2 + m_3 \Rightarrow f(a,b) = \overline{m_2} \cdot \overline{m_3} = M_1 \cdot M_0 \Rightarrow \underline{d}$

③ $f(a,b) = \overline{\bar{a}(b + \bar{a}\bar{b})} = \overline{\bar{a}(b + \bar{a})(b + \bar{b})} = \overline{\bar{a}(b + \bar{a})} =$

$= \overline{\bar{a}b + \bar{a}\bar{a}} = \overline{\bar{a}(1 + b)} = \bar{\bar{a}} = ab + a\bar{b} = m_3 \cdot m_2$

$\overline{f(a,b)} = m_0 + m_1 \Rightarrow f(a,b) = \overline{m_0 + m_1} = \overline{m_0} \cdot \overline{m_1} = M_3 \cdot M_2$

$f(a,b) = \prod(2,3) \Rightarrow b$

④ $f(a,b) = \overline{\bar a\,(b + \bar a \bar b)} \cdot \overline{\bar a\,(\bar a + b)(b + \bar b)} = \overline{\bar a\,(\bar a + b)} =$

$= \overline{\bar a \bar a + \bar a b} = \overline{\bar a\,(1 + b)} = a = a b + a \bar b = m_2 + m_3$

$f(a,b) = \Sigma\,(2,3) \Rightarrow \underline{\underline{b}}$

⑤ $f(a,b) = \overline{(\bar a + a \bar b)} = \overline{(\bar a + a)(\bar a + \bar b)} = \overline{\bar a + \bar b} = a b = m_3$

$f(a,b) = \Sigma\,(3) \Rightarrow \underline{\underline{c}}$

⑥ $f(a,b) = \overline{\bar a + a \bar b} = m_3 \Rightarrow \overline{f(a,b)} = m_0 + m_1 + m_2 \Rightarrow$

$f(a,b) = \overline{m_0 + m_1 + m_2} = \overline{m_0} \cdot \overline{m_1} \cdot \overline{m_2} = M3 \cdot M2 \cdot M1$

$f(a,b) = \Pi\,(1,2,3) \Rightarrow \underline{\underline{a}}$

⑦ $f_1(a,b) = \Sigma\,(0,3)$

$f_2(c,d) = \Pi\,(1,2)$

$g(a,b,c,d) = f_1(a,b) \oplus f_2(c,d)$

$f_1 = m_0 + m_3 = \bar a \bar b + a b$

$f_2 = M1 \cdot M_2 \Rightarrow \overline{f_2} = M_0 + M_3 \Rightarrow f_2 = \overline{M_0 + M_3} = \overline{M_0} \cdot \overline{M_3}$

$f_2 = m_3 \cdot m_0 = c d + \bar c \bar d$

$g = (\bar a \bar b + a b) \oplus (c d + \bar c \bar d) = \overline{(\bar a \bar b + a b)}\,(c d + \bar c \bar d) + (\bar a \bar b + a b)\,\overline{(c d + \bar c \bar d)} =$

$= (\overline{\bar a \bar b} \cdot \overline{a b})(c d + \bar c \bar d) + (\bar a \bar b + a b)(\overline{c d} \cdot \overline{\bar c \bar d}) =$

$= (a + b)(\bar a + \bar b)(c d + \bar c \bar d) + (\bar a \bar b + a b)(\bar c + \bar d)(c + d) =$

$= (\cancel{a \bar a} + a \bar b + \bar a b + \cancel{b \bar b})(c d + \bar c \bar d) + (\cancel{c \bar c} + \bar c d + c \bar d + \cancel{d \bar d})(\bar a \bar b + a b) =$

$= \underbrace{a \bar b c d}_{m_{11}} + \underbrace{a \bar b \bar c \bar d}_{m_8} + \underbrace{\bar a b c d}_{m_7} + \underbrace{\bar a b \bar c \bar d}_{m_4} + \underbrace{\bar a \bar b \bar c d}_{m_1} + \underbrace{\bar a \bar b c \bar d}_{m_2} + \underbrace{a b \bar c d}_{m_{13}} + \underbrace{a b c \bar d}_{m_{14}}$

$g = \Pi\,(1,2,4,7,8,11,13,14) \Rightarrow \underline{\underline{c}}$

(8) $A = (a_1, a_0) \times B = (b_1, b_0) \Rightarrow P(p_3, p_2, p_1, p_0)$

a_1	a_0	b_1	b_0	p_3	p_2	p_1	p_0
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0
0	0	1	0	0	0	0	0
0	0	1	1	0	0	0	0
0	1	0	0	0	0	0	0
0	1	0	1	0	0	0	1
0	1	1	0	0	0	1	0
0	1	1	1	0	0	1	1
1	0	0	0	0	0	0	0
1	0	0	1	0	0	1	0
1	0	1	0	0	1	0	0
1	0	1	1	0	1	1	0
1	1	0	0	0	0	0	0
1	1	0	1	0	0	1	1
1	1	1	0	0	1	1	0
1	1	1	1	1	0	0	1

$p_3 = 1 \leftrightarrow A \times B > 8$

$\Downarrow$

$\left.\begin{array}{l} A = 3 \\ B = 3 \end{array}\right\} 9$

$p_3 = \Sigma(15) \Rightarrow \underline{\underline{d}}$

(9) Probleme 8 sacar función canónica de p1

$p_1 = \Sigma(6, 7, 9, 11, 13, 14) \Rightarrow \underline{\underline{a}}$

(10) Probleme 8 sacar función canónica de p0

$p_0 = \Sigma(5, 7, 13, 15) \Rightarrow \underline{\underline{c}}$

(11) Probleme 8 fun. can. p_2

$p_2 = \Sigma(10, 11, 14) \Rightarrow \underline{\underline{d}}$